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CORRELATION MEASUREMENTS IN THE
BOUNDARY LAYER OF A FLAT PLATE

WILLIAM L. MARTIN III
AND
ARTHUR T. WHITE

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CORRELATION MEASUREMENTS IN THE
BOUNDARY LAYER OF A FLAT PLATE

by

William L. Martin III
Lieutenant, U.S. Navy
Nav. E., Massachusetts Institute of Technology (1954)

Arthur T. White
Lieutenant Commander, U. S. Navy
B.S., Massachusetts Institute of Technology (1948)

Submitted in Partial Fulfillment
of the Requirements for the
Degree of Master of Science

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

July, 1958

Cambridge, Massachusetts
July 21, 1958

Professor Leicester F. Hamilton
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Sir:

In accordance with the requirements of the degree
of Master of Science, we herewith submit a thesis entitled
"Correlation Measurements in the Boundary Layer of a Flat
Plate."

ABSTRACT

CORRELATION MEASUREMENTS IN THE

BOUNDARY LAYER OF A FLAT PLATE

by

WILLIAM L. MARTIN, III, LIEUTENANT, U. S. NAVY

ARTHUR T. WHITE, LIEUTENANT COMMANDER, U. S. NAVY

Submitted to the Department of Nuclear Engineering
on July 21, 1958, in partial fulfillment of the requirements for the degree of Master of Science.

The objective of this thesis was to assemble an instrumentation system and make limited measurements of correlation quantities in the boundary layer of a flat plate with an unheated starting length.

A system consisting of suitable probes, amplifier, calibration and measuring circuits, and voltmeter was assembled and tested. A series of measurements of the characteristic quantities of turbulent fluid flow were made by a traverse through the boundary layer at one location on the plate.

Although there is considerable scatter in the results, the system can be expected to give satisfactory results after the recommended equipment modifications and test procedures are effected.

Thesis Supervisor: R. J. Nickerson
Title: Assistant Professor of Mechanical Engineering

ABSTRACT

CONVERSION MECHANISM IN THE

ACQUANT LAYER OF A FLAT PLATE

by

WILLIAM T. MARTIN, III, LIEUTENANT, U. S. NAVY

WILLIAM T. MARTIN, LIEUTENANT COMMANDER, U. S. NAVY

Submitted to the Department of Physics, Massachusetts Institute of Technology, in partial fulfillment of the requirements for the degree of Doctor of Science.

The objective of this thesis was to establish an instrumentation system and make limited measurements of conversion quantities in the boundary layer of a flat plate with an upstream acoustic input.

A system consisting of multiple probes, amplifier, calibration and recording circuits, was designed and assembled and tested. A series of measurements of the conversion quantities of turbulent flow was made by a traverse through the boundary layer at one location on the plate.

Although there is considerable scatter in the results, the system was found to give satisfactory results after the recommended equipment modifications and test procedures are followed.

Thesis Supervisor: R. J. Scheraga
Assistant Professor of Mechanical Engineering
Title:

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1. INTRODUCTION

In the theory of turbulent fluid flow and heat transfer, the correlated quantities \overline{uv} and \overline{vt} have particular significance. The simplified equations that describe the boundary layer over a heated flat plate may be written as: (See Appendix B for derivation)

Momentum transfer:

$$\frac{\partial}{\partial x} (\bar{P} + \rho \bar{u}^2 + \rho \bar{u}^2) = \frac{\partial}{\partial y} (\mu \frac{\partial \bar{u}}{\partial y} - \rho \overline{uv} - \rho \overline{uv}); \quad (1)$$

Energy transfer:

$$\frac{\partial}{\partial x} (\bar{uT} + \bar{ut}) = \frac{\partial}{\partial y} (k \frac{\partial \bar{T}}{\partial y} - \overline{vT} - \overline{vt}). \quad (2)$$

Analytical solution of these equations is possible only under very restrictive conditions. For such solutions the following definitions of the eddy shear stress coefficient, ϵ_m and the eddy energy transfer coefficient, ϵ_h are introduced:

$$\text{and} \quad \overline{uv} = -\epsilon_m \frac{\partial \bar{u}}{\partial y} \quad (3)$$

$$\overline{vt} = -\epsilon_h \frac{\partial \bar{T}}{\partial y} \quad (4)$$

Assumed analogies between these coefficients together with suitable boundary conditions has been an accepted method for solving problems for a great many years. Very little work has been done, however, to actually measure \overline{uv} and \overline{vt} and experimentally substantiate the theory.

The literature contains little data other than that of Johnson in ref. 1. He obtained measurements of these and other quantities in the

boundary layer over a flat plate with an unheated starting length. His results are given for only one vertical exploration of the boundary layer, and they agree with the expected behavior of \overline{uv} and \overline{vt} as functions of distance from the heated plate.

It has been proposed that a thorough investigation of the thermal boundary layer properties be undertaken at M.I.T. Therefore, it is of fundamental importance to design an instrumentation system of sufficient sensitivity and accuracy to evaluate such properties.

It was the purpose of this thesis to assemble such an instrumentation system and make preliminary measurements over a heated flat plate. Theoretically a hot-wire anemometer system will provide sufficient data for calculating \overline{uv} and \overline{vt} . A basic system was assembled and tested. Certain components were available in the laboratory, and the remaining items were selected from commercially available instruments. Appendix D contains a description of the equipment.

and they agree with the expected behavior of the two in the future.

It has been proposed that a strongly ionized gas, such as a plasma, is a good medium for the propagation of electromagnetic waves. The purpose of this paper is to study the propagation of electromagnetic waves in a plasma. The plasma is assumed to be a uniform, isotropic medium. The wave is assumed to be a plane wave. The plasma is assumed to be a uniform, isotropic medium. The wave is assumed to be a plane wave. The plasma is assumed to be a uniform, isotropic medium. The wave is assumed to be a plane wave.

It was the purpose of this thesis to assess the need for instrumentation systems and their preliminary requirements for a manned star plane. Theoretically a two-star measurement system will provide sufficient data for calculating \bar{v} and \bar{v}_T . A basic system was established and tested. Certain components were available in the laboratory, and the remaining items were selected from commercially available instruments. Appendix 3 contains a description of the instrumentation.

II. PROCEDURES

The experiment was conducted in the Heat Measurement Laboratory using one of the small scale wind tunnels. The boundary layer was developed over a 40" aluminum plate resting on the bottom of the tunnel. The last twelve-inch section of the plate was heated by a coil of resistance heating wire on the underside. Measurements were made along a vertical traverse through the boundary layer ten inches from the start of the heated section.

The hot-wire anemometer equipment consisted of hot-wire probes, a Flow Corporation HWB amplifier and bridge unit and a Ballantine 320 true root-mean-square volt-meter.

Two different hot-wire anemometer probes were used. One was a single-wire; the other a V-wire probe. Appendix D describes the equipment in greater detail.

Because the V-probe soon proved unacceptable, the single-wire probe was used thereafter. Orienting the wire first at plus 45° , then at minus 45° to the free stream velocity gave the desired data. Since only the time-averaged quantities enter the calculations, data for each orientation did not need to be taken at the same time.

After thermal aging for a half-hour, the wire was carefully calibrated by recording the wire current at each resistance ratio for various wind velocities. A micromanometer was used to obtain accurate velocities.

A plot of I^2 vs \sqrt{U} appears in Fig. 8 for $\gamma = 1.2$ and 1.4. These

11. APPARATUS

The experiment was conducted in the High Temperature Laboratory using one of the small cells used usually. The specimen layer was supported over a 50' aluminum plate resting on the bottom of the vessel. The low resistance section of the plate was heated by a coil of resistance heating wire on the underside. Measurements were made along a vertical traverse through the furnace layer and higher from the start of the heated section.

The hot-wire resistance element consisted of two-wire probes, a fine tungsten wire welded to bridge wire and a resistance 350 ohm tungsten-wire probe.

The following hot-wire resistance probes were used. One was a single-wire probe and a 5-wire probe. Appendix 9 describes the equipment in greater detail.

Between the 5-probe and single-wire probe, the single-wire probe was used. During the time that it was 47°, then at about 47° to the time it was 47° the heated wire. From only the time-averaged resistance with the probe, data for each observation did not need to be taken at the same time.

After several days for a full-day, the wire was carefully calibrated by recording the wire current at each resistance value for various wire resistances. A microammeter was used to obtain accurate resistance. A plot of I^2 vs R appears in fig. 2 for 1.1 and 1.2. These

curves agree with the characteristic straight lines predicted by the King equation.

From these curves it is possible to calculate I' from equation (36), where

$$I' = \frac{C_2}{4I \sqrt{U}} \quad (5)$$

As equation (72) predicts, the quantities \overline{uv} and \overline{vt} are related by

$$\frac{1}{2} \left(\frac{i_s}{I'} \right)^2 \left[\left(\frac{M_{sn}^2 - M_n^2}{M_{scn} - M_{sn}^2} \right)_b - \left(\frac{M_{sn}^2 - M_n^2}{M_{scn} - M_{sn}^2} \right)_a \right] \\ = - \overline{uv} + \left\{ \frac{\sqrt{2}}{4I'} \frac{\alpha e}{1 - [1 + \alpha_e(\overline{T}_g - \overline{T}_e)]} \right\} (I_a + I_b) \overline{vt}, \quad (6)$$

where the subscripts a and b denote the quantities for the two wire orientations. It is important that the wires of whatever probe is used be correctly angled to the flow, i.e. $\pm 45^\circ$ for this case.

The procedure for measuring and calculating these quantities follows:

Ref. 12, the HWB Instruction Book, describes how to compute the square-wave current and how to set the desired resistance ratios. Briefly, the "Bridge Null" dial of the HWB is adjusted to balance the bridge for a "cold" current in the wire of about one and a half milliamps.

At one of the available resistance ratios, the current is increased in the wire until the bridge is again balanced. The values of resistance ratio and "bridge null" dial setting then enter the calculation of square-wave current as shown in Appendix E.

The following sequence of readings was obtained at the maximum tunnel velocity of 26 ft/sec with the plate first unheated, then heated:

curves agree with the characteristic straight lines predicted by the King equation.

From these curves it is possible to calculate V_1 from equation (3b),

where

$$(3) \quad V_1 = \frac{V_2}{\sqrt{1 + \frac{V_2^2}{V_1^2}}}$$

In equation (3b) predicted, the quantities V_1 and V_2 are related by

$$(4) \quad \frac{V_1}{V_2} = \frac{\left(\frac{V_1}{V_2} \right)^2 \left(\frac{M_{12}}{M_{22}} - \frac{M_{11}}{M_{21}} \right) - \left(\frac{M_{12}}{M_{22}} - \frac{M_{11}}{M_{21}} \right)}{\left(\frac{V_1}{V_2} \right)^2 \left(\frac{M_{12}}{M_{22}} - \frac{M_{11}}{M_{21}} \right) - \left(\frac{M_{12}}{M_{22}} - \frac{M_{11}}{M_{21}} \right)}$$

where the subscripts 1 and 2 denote the quantities for the two wire orientations. It is important that the wires of whatever shape is used be correctly aligned to the flow, i.e. $\frac{1}{2} \pi$ for this case.

The procedure for measuring and calculating these quantities follows:

Ref. 12, the King Instruction Book, describes how to compute the square-

wave current and how to set the desired resistance ratios. Briefly, the

"Bridge Null" dial of the flow is adjusted to balance the bridge for a

"cold" current in the wire of about one and a half milliamperes.

As one of the available resistance ratios, the current is increased in

the wire until the bridge is again balanced. The values of resistance ratio

and "Bridge Null" dial setting then enter the calculation of square-wave

current as shown in Appendix E.

The following sequence of readings was obtained at the maximum tunnel

velocity of 10 ft/sec with the plate first unstested, then heated:

With the wire at plus 45° , values of M_{sn} , M_{scn} and I_a were recorded at resistance ratios of 1.2 and 1.4. This was repeated at predetermined distances, y , above the plate as measured by the micro-traverse gear, starting where the probe shank touched the plate.

Values of M_{sn} , M_{scn} and I_b were subsequently recorded with the wire at minus 45° for the same values of y .

The purpose of the hot and cold plate readings becomes clear when one assumes $t = 0$ for the unheated plate. Then equation (72) is

$$\overline{uv} = -\frac{1}{2} \left(\frac{I_s}{I_t} \right)^2 \left[\left(\frac{M_{sn}^2 - M_a^2}{M_{scn}^2 - M_{sn}^2} \right)_b - \left(\frac{M_{sn}^2 - M_a^2}{M_{scn}^2 - M_{sn}^2} \right)_a \right]. \quad (7)$$

However, with the heated plate the full equation applies.

The remaining part of the coefficient of \overline{vt} , $\frac{\alpha_e}{\gamma - [1 + \alpha_e(T_g - T_e)]}$, may be calculated as shown in Appendix E.

Making measurements at two resistance ratios gives two simultaneous equations at each y . Solution of these equations gives \overline{uv} and \overline{vt} for the hot plate, and \overline{uv} for the cold plate. With the plate cold \overline{uv} is over-determined, so the average of the two equations was taken.

A traverse of the boundary layer with the wire perpendicular to the mean flow gives a comparison of the wire current with that in the free stream. From the I^2 vs \sqrt{U} calibration data on the wire, the average velocity as a function of y can be picked off.

Similarly, a measurement of wire currents at a position first with the plate heated, then unheated, allows calculation of the temperature rise above ambient by the method of Appendix E.

with the value of β as β_{max} , values of β_{min} and β_{opt} were determined
of maximum value of 1.1 and 1.4. This was repeated at predetermined
distances, β above the head as measured by the micro-thermometer,
measuring above the water about 100 cm. from the plate.
Values of β_{min} , β_{opt} and β_{max} were independently compared with the value
as given in β^0 for the same values of γ .

The curves of the head and side plate readings become clear when one
inserts a β for the observed value. This equation (7) is

$$(7) \quad \overline{w} = -\frac{1}{2} \left(\frac{1}{\beta} \right) \left[\left(\frac{1}{\beta} - \frac{1}{\beta^0} \right) \left(\frac{1}{\beta} - \frac{1}{\beta^0} \right) \right]$$

However, when the value of β is substituted in the side plate equation
The resulting curve of the coefficient of β , β^0
may be calculated as shown in Appendix A.

Having measurements of the maximum values gives two simultaneous
equations in β . Values of these equations gives β and β^0 for the
two plates, and β for the side plate. The value of β is over-
estimated, as the average of the two equations was taken.

A diagram of the boundary layer with the side plate divided in the
two parts gives a comparison of the side plate with that in the first
diagram. The two β or β^0 values are then on the side, the average
value of β as a function of γ can be plotted.

Finally, a comparison of side curves at a positive first with
the side curve, the minimum value calculation of the temperature rise
above values by the method of Appendix A.

III. RESULTS

A. Figure 1 shows \overline{uv}/U_∞^2 as a function of vertical distance from the plate for the heated and unheated conditions. Reasonable agreement with published data is demonstrated. The value of this quantity is greater in the heated plate condition than for the cold plate, as was noted in reference (1).

Figure 2 is a graph of the temperature $(\overline{T}_g - T_e)/(T_s - T_e)$ and velocity (\overline{U}/U_∞) profiles across the boundary layer.

Figure 3 is a plot of \overline{uv}/U_∞^2 and $\overline{vt}/U_\infty (T_s - T_e)$ for the heated plate. The best data obtained is shown. The extreme scatter of the data precludes computing E_m and E_h as functions of y in order to establish an analogy between them.

B. The basic instrumentation selected includes: (a) a hot wire probe with 0.00015" tungsten wires; (b) a Flow Corporation, Hot Wire Anemometer, model HWB; and (c) a modified Ballantine Laboratory, True RMS Voltmeter, model 320. This equipment will measure accurately the properties of a momentum boundary layer. It is probable that the additional properties of a thermal boundary layer can also be detected with this instrumentation; however, this cannot be stated conclusively. Appropriate recommendations are included in section VI. The equipment is illustrated in figure 4.

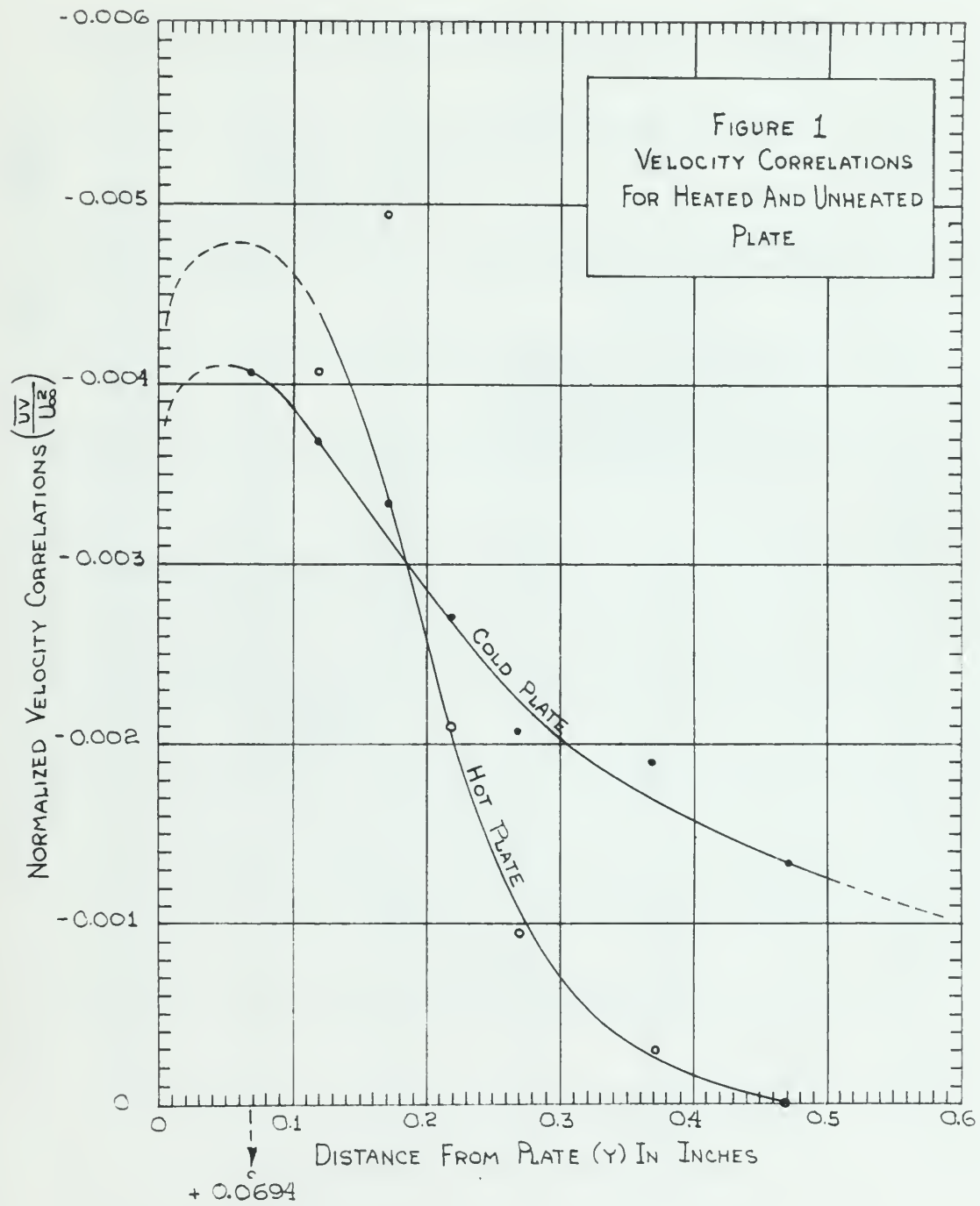
III. RESULTS

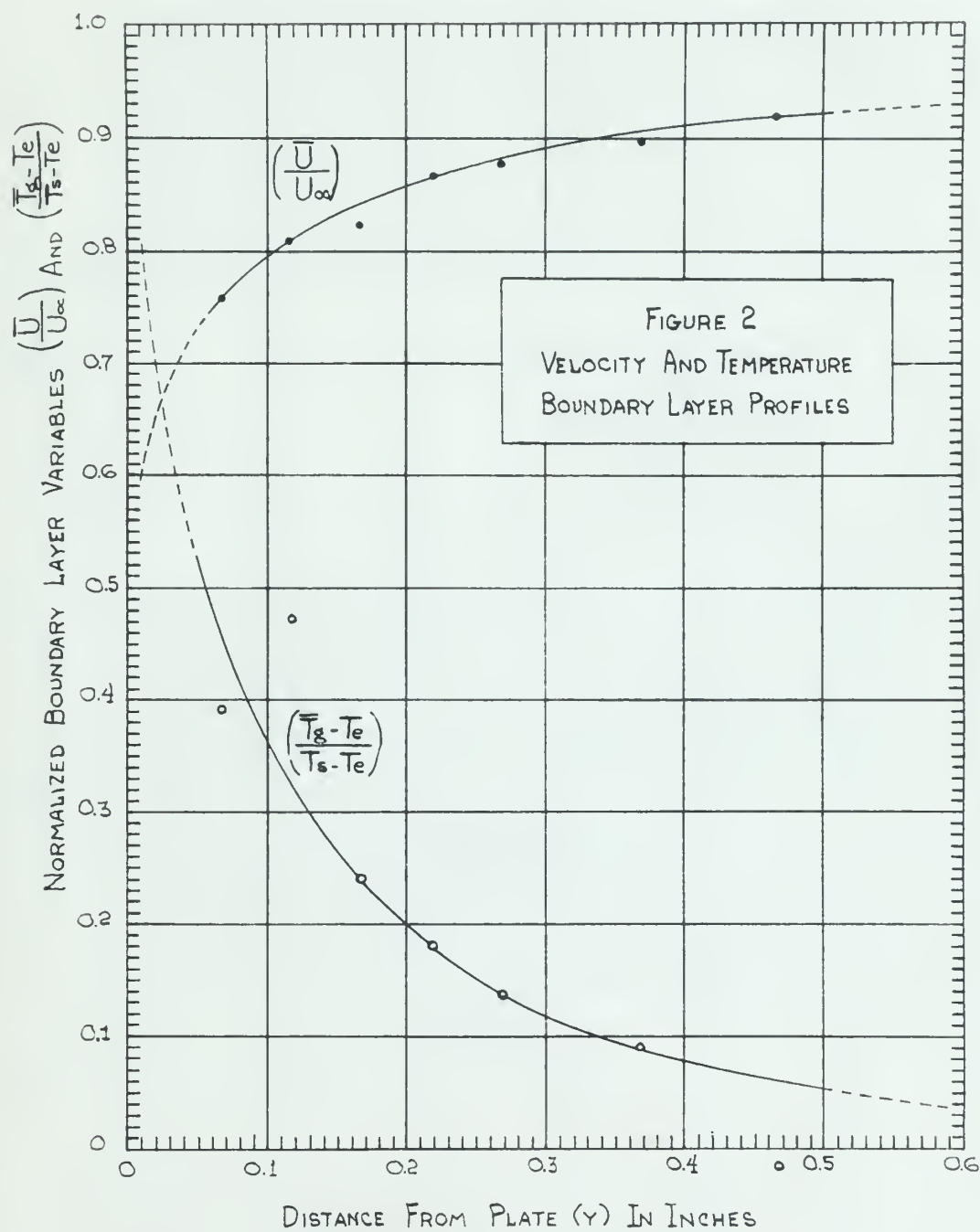
1. Figure 1 shows \bar{v}_{100} as a function of reduced distance from the plate for the heated and insulated conditions. Reasonable agreement with published data is demonstrated. The value of this quantity is greater in the heated than insulated case for the cold plate, as was noted in reference (1).

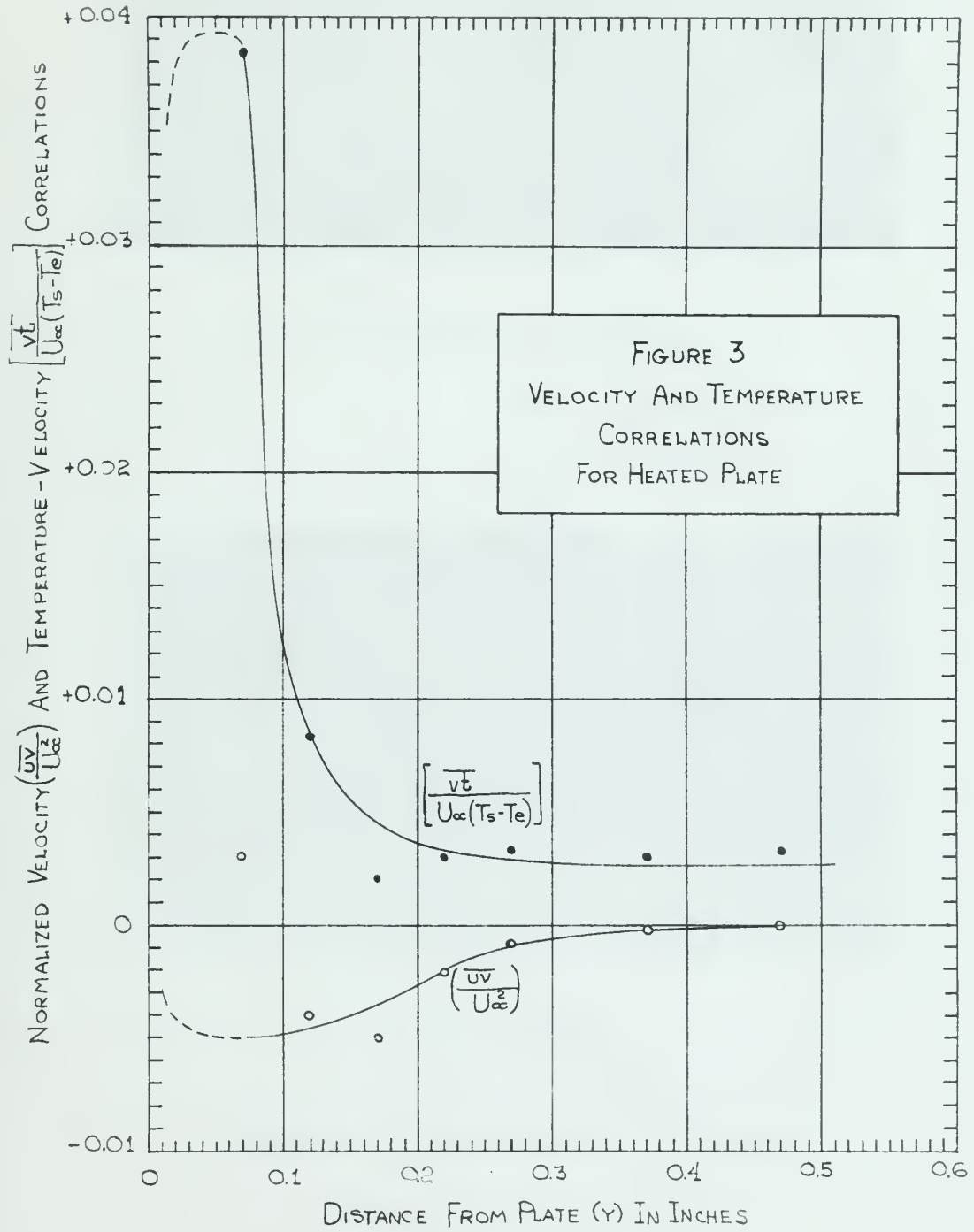
Figure 2 is a graph of the temperature $(T-T_0)/(T_1-T_0)$ and velocity (U/U_0) profiles across the boundary layer.

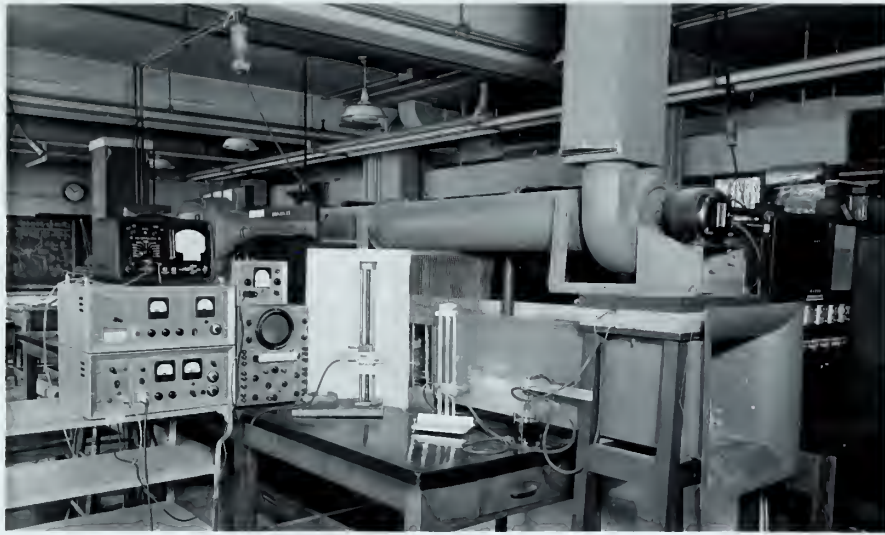
Figure 3 is a plot of \bar{v}_{100} and \bar{v}_{100}^* for the heated plate. The data obtained in this case are shown. The various symbols of the data included computing \bar{v}_{100} and \bar{v}_{100}^* as functions of \bar{v}_{100} as indicated in analogy between them.

2. The basic heat-transfer-related results: (a) a hot wire probe with 0.00125 inch diameter; (b) a thin film sensor; and (c) a thermocouple, are shown in Figures 4 and 5. This comparison will suggest the possibility of a common boundary layer. It is probable that the additional properties of a thermal boundary layer can also be associated with this boundary layer, but cannot be stated conclusively. Appropriate recommendations are included in section IV. The symbols as illustrated in Figure 4.









(a). Equipment Arrangement



(b). Probes

Figure 4 - Experimental Equipment



IV. DISCUSSION OF RESULTS

A. The plot of \overline{uv}/U_∞^2 in Fig. 1 has about the same order of magnitude, although somewhat higher, as that shown by Johnson in ref. 1. The slope of this curve is much greater than Johnson's however. He did obtain two or three points close to the heated plate that are much higher than those for the cold plate. Further from the plate Johnson's velocity correlation quantities are almost identical for the heated and unheated conditions.

On the other hand, the quantities shown in Fig. 1 for the heated plate remain generally higher than for the cold plate over the span measured. This difference could easily be due to experimental error, however there is a possible explanation. Thermal convection from the heated plate could be expected to disrupt the laminar sublayer causing some increase in the turbulence of the flow close to the plate. Fluid moving from the sublayer would have lower velocities and therefore tend to correlate $a(-u)$ with $a(-v)$ to add to the quantity $(-\overline{uv})$ for the cold plate.

B. The plots of \overline{U}/U_∞ and $(T_g - T_e)/(T_s - T_e)$ as a function of distance from the plate show the typical characteristics of these functions. The velocity profile indicates that measurements should be taken closer to the plate to further delineate the shape of the curve. To do this a much shorter hot-wire would be needed.

IV. MINIMUM OF ERROR

A. The plot of $\overline{u^2}/V_0^2$ in Fig. 1 was shown the same order of magnitude, although somewhat higher, as that shown by Johnson in ref. 1. The slope of this curve is much greater than Johnson's however. He did obtain two or three points closer to the heated plate than any other than those for the cold plate. Further from the plate Johnson's velocity measurements quantities are almost identical for the heated and unheated conditions. On the other hand, the quantities shown in Fig. 1 for the heated plate remain generally higher than for the cold plate over the span measured. This difference could easily be due to experimental error, however there is a possible explanation. Thermal convection from the heated plate could be expected to disturb the laminar velocity causing some increase in the turbulence at the plate and the plate. This moving from the surface would have lower velocities and therefore tend to correlate $u(-y)$ with $u(y)$ so that the quantity $(-uv)$ for the cold plate.

B. The plots of $\overline{u^2}/V_0^2$ and $(T-T_0)/(T-T_0)$ as a function of distance from the plate show the typical characteristics of these functions. The velocity profile indicates that measurements should be taken closer to the plate to further determine the shape of the curve. To do this a much steeper slope would be needed.

Extrapolating the curve of $T_g - T_e$ in Fig. 10 to the plate gave an estimated plate surface temperature, T_s , of 8°C above ambient. Such a low temperature difference made temperature measurements difficult.

Another more accurate method of measuring temperature would be to record the bridge null at "cold current" (about one and a half milliamps) while traversing the boundary layer. For this a bridge galvanometer amplifier is required to enable more accurate bridge null settings than is possible with this model HNE. The later HNE models do incorporate such an amplifier.

C. As stated in the Results, the scatter in the computed points $\overline{v_t}/U_\infty (T_s - T_e)$ in Fig. 3 did not warrant calculation of E_m and E_h by equations (3) and (4). This is one of the most important ultimate goals of these measurements that was not realized in this thesis. No reliable curve of $\overline{v_t}$ can be inferred, and several attempts to deduce better values of this quantity produced no better results than those shown. A higher plate temperature would likely give larger, more-easily-measured quantities.

The data does not show the predicted behavior of \overline{uv} and $\overline{v_t}$ going to zero at the plate. At the closest approach, the center of the wire is still 0.068 inches from the plate. Any decrease in these quantities must therefore be inside this distance.

D. The equations (69) and (70) in Appendix B for two wires apply also for wires mounted on an X-probe. Johnson used an X-probe with both wires hot for detecting velocity fluctuations and a single cold wire close by for temperature variations. The hot-wire theory predicts that it is unnecessary to use more than one hot-wire. Since equation (71) has five unknowns, it is necessary only to make measurements with one wire at five resistance ratios and solve the resulting equations simultaneously.

If two wires on an X-probe are controlled independently by two HWB's with outputs measured on separate meters, equations (69) and (70) may be solved together at five resistance ratios to yield the boundary layer properties. The two important correlated quantities are dominant in equation (72).

The practice in this experiment of orienting the probe at $\pm 45^\circ$ simulated an X-probe and eliminated the problem of matching the two wires to close tolerances. Experience with the V-probe supports the comment made by Johnson (p. 54 of ref. 1) that it is unlikely I' , I , α_e , the time constant, length, diameter and other characteristics of two wires can ever be matched simultaneously.

A few comments on the V-probe may warn others of the pitfalls of its peculiarities.

It was first discovered that the resistance of the common current lead and center needle is unavoidably in one arm of the bridge circuit. Since the voltage across the center lead is proportional to the sum of the two wire currents, the bridge null for one wire is affected by current in the other.

The second problem is serious, particularly in a thin boundary layer. The two wires are sufficiently separated to be in different stream conditions and do not correlate quantities at a point.

The third problem has been discussed - that of matching any two wires.

Finally, several times it was noted that wires on the V-probe were not taut and straight several hours after rewiring. This means that wires are not at the specified angle to the stream.

E. One problem that detracts from the accuracy of the results is ambient temperature variations while the measurements are being made in a traverse

It was found that the temperature was controlled independently by the two
with separate heaters for separate wires, equations (5) and (7) say
which together with five resistance ratios in the boundary layer
properties. The two important mechanical quantities are defined in
equation (7).

The analysis in this paper is of a system of wires at $\frac{1}{2}$ inch
laced at 1-inch and estimated the position of the wires to
give a reasonable approximation. The 1-inch wires are the most
by Johnson (p. 34 of ref. 1) that it is possible to have the
contact, length, diameter, and other characteristics of the wires can vary
in a certain range.

A few comments on the 1-inch wires can be made on the basis of the
properties.

It was first discovered that the resistance of the common circuit
long and center wires is considerably in excess of the bridge circuit.
Since the voltage across the contact is proportional to the sum of
the two wire resistances, the bridge will not be affected by current
in the other.

The second problem is serious, particularly in a thin boundary layer.
The two wires are sufficiently separated so as to be in different stream condi-
tions and do not represent quantities at a point.

The third problem has been discussed - that of reading any two wires.
Finally, a fourth point is mentioned that arises on the 1-inch wires
and that are outside several wires after reading. This means that there
are not as the specified ratio in the system.

6. One problem that arises from the accuracy of the results is related
temperature variations while the measurements are being made in a stream

through the boundary layer. Using the minimum number of two resistance ratios at each y , it required four to six hours to take data at seven points. Since the ambient temperature can change by several degrees during this time, it is desirable to have a constant-temperature air input to the tunnel.

It was noticed that there was considerable velocity fluctuation in the free stream on the axis of the tunnel. This masked the "edge" of the momentum boundary layer.

To reduce the above defects of this tunnel it is desirable that the entrance be fitted with a plenum where the air can be heated a few degrees above the ambient at the same time the velocity fluctuations are being smoothed.

Along with tunnel entrance modifications other test facility improvements should be incorporated.

Although a complete survey of the boundary layer should include measurements at various distances along the plate, it would be well for the uninitiated to practice in boundary layers at least two inches thick. This means having an unheated starting length of at least 40 inches long and a heated section 30 inches long. The tunnel should be three to four feet wide to approximate an infinitely wide plate.

The heated plate should be well-insulated on the bottom and equipped with heating coils and thermocouples for obtaining a constant (q/Λ) or constant temperature plate as desired.

F. Of all the problems encountered in this experiment, that of measuring accurate (i.e. less than 5%) true root-mean-square voltages caused the most consternation. The best available information indicated that the Ballantine Model 320 and Flow Corporation TBM were the only two meters commercially available reputed to measure RMS voltages on a random signal.

When the short time constant of the Model 320 was lengthened, it became the most reliable instrument. However, both meters exhibited the annoying characteristic of indicating different values of the same voltage on adjacent scales. The TBM became unusable for this reason, and the Model 320 gave ambiguous readings only at times. The limited "crest factor" of the Model 320 could permit peak clipping of the signal on the lower scale and thereby alter the waveform. However, such should not be the case for the TBM with its much higher "peak factor."

This problem was not resolved and is the most likely cause of the wide scatter in the calculated results. Knowing that this problem existed provided the hint to smooth the data as a function of y before re-calculating \overline{uv} and \overline{vt} . Even so the improvement was not marked.

It is interesting to note that the literature reviewed for this thesis makes no mention of true RMS measurements or time-averaged readings in similar work.

The Model 320 could be improved by making a well-designed permanent modification to obtain a long time constant, mean square averaging circuit. If the meter time constant cannot be suitably controlled, then a five to ten minute time recording of the output could be graphically integrated to establish the best value of each voltage.

A more elaborate scheme of calibrating the meter is needed. The most appropriate method would be to use a variable-frequency generator with a calibrated output adjustable from one millivolt to one volt. It could be used to resolve the problem of peak clipping of the signals.

G. One test of accurate data is its repeatability. Time permitted little experimenting along this line, but it was often noted that the bridge could be rebalanced and currents reset to a maximum variation of one part in 150 or 0.7%.

When the short time constant of the input 320 was investigated, it became the most reliable instrument. However, high speed exhibited the varying characteristics of indicating different values of the same voltage on adjacent scales. The two scales available for this reason, and the input 320 gave ambiguous readings only at times. The first "approximate" of the input 320 could result from clipping of the signal on the lower scale and thereby alter the waveform. However, such should not be the case for the first with the much higher "approximate".

That output was not verified but is the more likely cause of the many errors in the calculated results. Knowing that this problem existed provided the data as shown in the table as a function of γ before re-calculating the data. Even so the improvement was not sufficient.

It is interesting to note that the literature reviewed for this thesis makes no mention of any of the measurements or time-constant problems in similar work.

The input 320 could be improved by making a self-designed instrument modification to provide a long time constant, with some averaging circuit. If the input time constant cannot be suitably modified, then a time to for about the resolution of the output could be practically impossible to establish the best value of each voltage.

A more elaborate scheme of calibrating the input is needed. The most appropriate method would be to use a variable-frequency generator with a calibrated output adjustable from one millivolt to one volt. It could be used to measure the problem of beam clipping of the signal.

6. The best of accurate data is the reproducibility. Two separate trials were made along the line, but it was often noted that the bridge could be recalibrated and output varied in a similar manner as one part in 100

In the presence of signal fluctuations the galvanometer fluctuated also. This was particularly noticeable at high resistance ratios. In setting average currents it became necessary to average the needle fluctuations by eye. Occasionally this led to variations of one part in 75 or 1.3%. Provision of a selectable time constant on the galvanometer would assist in setting more accurate currents and resistance ratios.

At the time they were taken, meter readings could be averaged within 2% in spite of the slow fluctuations. However, it was rarely possible to repeat the data at a point because ambient conditions had changed. On the few occasions when it was possible to check results, meter readings were well within 5% barring anomalies already discussed.

In the present of rapid fluctuations the Government is situated also.

This has particularly noticeable of high business cycles. In setting
 over the course of years, however, it is necessary to consider the fluctuations by
 eye. Consequently this has to be taken into account of the fact that in 1931, 1932-
 1933, and 1934 the fluctuations have been on the whole somewhat smaller in
 relation to the average than in the previous period.

At the same time, however, the fluctuations could be stronger within
 the limits of the fluctuations. However, it has to be taken into
 account the fact that the fluctuations are somewhat smaller and stronger. On the
 one hand, it is not possible to make a general statement, which would mean
 that the fluctuations are always stronger.

It is not possible to make a general statement, which would mean
 that the fluctuations are always stronger.

It is not possible to make a general statement, which would mean
 that the fluctuations are always stronger.

It is not possible to make a general statement, which would mean
 that the fluctuations are always stronger.

It is not possible to make a general statement, which would mean
 that the fluctuations are always stronger.

It is not possible to make a general statement, which would mean
 that the fluctuations are always stronger.

It is not possible to make a general statement, which would mean
 that the fluctuations are always stronger.

V. CONCLUSIONS

Hot wire anemometer theory predicts that both of the desired correlation quantities, \overline{uv} and \overline{vt} , can be obtained with a single, matched, X-wire probe. Due to the extremely small volume affected by the fluctuation quantities at any instant, velocity-temperature sensing elements in a single probe is preferable to separate probe sensing elements.

The instrumentation system, modified as recommended in section VI, should yield data of sufficient accuracy for calculation of all the thermal boundary layer properties. The results presented do not affirm this statement without question, but they do indicate that further careful investigation is warranted. Refinement of instrument sensitivity and experimental technique are the paramount requisites.

7. DISCUSSION

For very low-frequency signals the value of the relative error, $\overline{\epsilon}$, can be calculated with a single, rather, X-ray probe, but in the frequency range covered by the frequency spectrum in our studies, relative error is calculated in a single probe is possible to separate probe signals.

The representation of the signal as presented in section VI, should yield data of sufficient accuracy for calculation of all the chemical boundary layer properties. The results presented in our studies this statement without question, but they do indicate that further careful investigation is warranted. Estimates of instrument sensitivity and experimental techniques are the permanent requirement.

VI. RECOMMENDATIONS

The following modifications of instrumentation and facilities are required for accurate measurements of the thermal boundary layer properties:

- (1) Permanent modification of the meter circuit of a Ballantine 320 to obtain a long time constant averaging period;
- (2) Accurate calibration of all the range scales on the voltmeter by use of an accepted standard frequency meter which has a calibrated voltage output;
- (3) Construction of an X-wire probe with the wires oriented accurately at 90° to each other and their resistance matched within 2%;
- (4) Construction of a longer heated section for the plate to increase the boundary layer thickness;
- (5) Construction of a larger scale wind tunnel with an inlet heated plenum chamber to insure a constant temperature, constant velocity air stream in the tunnel;
- (6) Procurement of an additional HWB, modified to include a sum and difference unit in lieu of the amplifier section.

In addition, the data processing techniques should be refined as follows:

- (7) An averaging technique must be used over and above that which is provided in (1) above - e.g., averaging of sequentially timed observations, or graphical integration of the recorded output of the voltmeter;
- (8) All five resistance ratios permitted by the wire size must be used

(6) All test specimens shall be prepared by the same method as used

the following:

specimens, or graphical representation of the recorded weight of

provided in (1) above - e.g., averaging of specimens) used in

(7) In averaging specimens used in test over and above that which is

in addition, the data representing specimens shall be reduced

difference will be less in the specified section.

(8) Treatment of an additional test, specified in Section 4, and not

are shown in the diagram.

(9) Specimens of a larger scale than those with no other means

and boundary layer specimens;

(a) Specimens of a larger scale section for the same or different

at 50 to each other and their respective weight and size.

(b) Construction of an 8-inch scale with the other material accordingly

weighting method;

use of an accepted standard frequency scale which has a calibrated

(c) Accurate calibration of all the range scales on the specimens of

to which a long time constant frequency meter.

(1) Treatment and calibration of the other specimens of a specimen of

properties:

prepared for accurate measurement of the linear boundary layer

the following conditions of measurement and facilities are

VI. CONCLUSIONS

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to calculate all five terms of equation # 7/ until their order of magnitude is firmly established. Programming of these equations for computer use would considerably speed data analysis.

to collect all the data of a system in order to
 separate it from the rest of the system. This is done
 by separating the data into two parts: the data
 which is used for the system and the data which is
 used for the rest of the system.

The first part of the data is the data which is
 used for the system. This is the data which is
 used for the system and the data which is used
 for the rest of the system.

The second part of the data is the data which is
 used for the rest of the system. This is the data
 which is used for the rest of the system and the
 data which is used for the system.

The third part of the data is the data which is
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 for the rest of the system.

The fourth part of the data is the data which is
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The fifth part of the data is the data which is
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The sixth part of the data is the data which is
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The seventh part of the data is the data which is
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The tenth part of the data is the data which is
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 which is used for the rest of the system and the
 data which is used for the system.

APPENDIX A

NOMENCLATURE

A	Cross-sectional area.
c	Heat capacity of wire.
C_p	Heat capacity of fluid at constant pressure.
C_1, C_2	Constants.
D	Wire diameter.
D^*	Wire diameter, units of 0.001 inch.
e	Voltage (general).
e_c	Output voltage from the amplifier due to square-wave current alone imposed on the wire.
e_{cn}	Output voltage from the amplifier due to square-wave current plus system noise.
e_n	Output voltage from the amplifier due to system noise.
e_s	Output voltage from the amplifier due to velocity fluctuations on the hot wire.
e_{sc}	Output voltage from the amplifier due to velocity fluctuations plus square-wave current.
e_{scn}	Same as e_{sc} plus system noise.
e_{sn}	Same as e_s plus system noise.
e_w	Hot-wire voltage before amplification.
F	Function expressing cooling effect on hot wire by velocity and pressure of stream.
F'	Defined as $\left. \frac{\partial F}{\partial V} \right _P$, the derivative of F with respect to velocity at constant P.
\bar{F}	Time-average value of F.

... ..

1	Time-average value of \bar{v}
2 $\frac{1}{T} \int_0^T v(t) dt$
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- f Instantaneous deviation value of F from \bar{F} .
- H Rate of heat loss from the hot wire per unit length.
- h Fluid enthalpy per unit mass.
- I Hot-wire DC current.
- \bar{I} Time-average of I .
- I' Defined as $\left. \frac{\partial I}{\partial v_p} \right|_p$, the derivative of hot-wire current with respect to velocity at constant pressure.
- i Instantaneous deviation value of I from \bar{I} .
- i_s Instantaneous deviation value of hot-wire current due to imposed square-wave.
- I_a, I_b Hot-wire DC current of wires a and b.
- J_1 Amplifier multiplier coefficient of e_w .
- J_2 Amplifier multiplier coefficient of time derivative of e_w .
- K_1 Meter constant.
- k Coefficient of thermal conductivity.
- L Length of hot-wire.
- M Meter reading of true RMS voltage.
- $M_c, M_{cn}, M_n, M_s, M_{sc}, M_{scn}, M_{sn}$ Meter readings corresponding to voltages, e , with same subscripts.
- N_{Pr} Prandtl number, equals $c_p \mu / k$.
- N_{Re} Reynolds number, equals $\mu \bar{U} D / \rho$.
- P Pressure of fluid in the boundary layer.
- \bar{P} Time-average of P .
- p Instantaneous deviation value of P from \bar{P} .
- P^* Pressure of fluid, units of atmospheres.
- q Rate of heat transfer per unit time.
- R Hot-wire resistance.
- \bar{R} Time-average of R .

[illegible]

r	Instantaneous deviation value of hot-wire resistance from \bar{R} .
R_e	Resistance of wire in equilibrium with ambient fluid temperature, T_e .
\bar{R}_e	Time-average value of R_e .
r_e	Instantaneous deviation value of R_e from \bar{R}_e .
R_g	Resistance of wire in equilibrium with fluid temperature, T_g .
\bar{R}_g	Time-average value of R_g .
r_g	Instantaneous deviation value of R_g from \bar{R}_g .
R_o	Resistance of wire at reference temperature, T_o .
T	Temperature.
\bar{T}	Time-average of T .
t	Instantaneous deviation value of T from \bar{T} .
T_e	Ambient temperature of the stream entering the tunnel.
T_g	Temperature of fluid at a point in the boundary layer.
\bar{T}_g	Time-average value of T_g .
T_o	An arbitrary reference temperature for computing resistance of wire material, handbook value.
T_s	Surface temperature of the heated plate.
T_w	Hot-wire temperature.
T	Temperature of the fluid in the free stream.
U	Velocity of the fluid in the x-direction.
\bar{U}	Time-average value of U .
u	Instantaneous deviation value of U from \bar{U} .
U	Velocity of the fluid in the free stream.
V	Velocity of the fluid in the y-direction.
\bar{V}	Time-average of V .
v	Instantaneous deviation value of V from \bar{V} .

[illegible]

V_A, V_B	Velocity of fluid perpendicular to wires A and B.
V_θ	Velocity of fluid at angle θ from perpendicular to wire.
V_p	General velocity perpendicular to wire.
v_p	General deviation value of V_p from its time-average.
V^*	Velocity of fluid in units of 100 ft/sec.
W	Velocity of fluid in x-direction.
\bar{W}	Time-average value of W .
W_1	Rate at which work is done on a particle of fluid due to external fields.
W_f	Rate at which work is done against viscous stresses.
X, Y, Z	Body forces in the x-, y-, and z-directions.
x, y, z	Distances in the direction of mean flow, perpendicular to the plate and transverse to the plate.
α_e	Temperature coefficient of resistivity at T_e .
α_g	Temperature coefficient of resistivity at T_g .
α_0	Temperature coefficient of resistivity at T_0 .
α	Thermal diffusivity, equals $k/\rho c_p$.
γ	Resistance ratio, resistance of hot wire to resistance of cold wire.
E_h	Coefficient of eddy heat (or energy) transfer.
E_m	Coefficient of eddy momentum transfer.
θ	Angle between perpendicular to wire and mean velocity vector.
μ	Viscosity.
ν	Kinematic viscosity.
ρ	Density.
τ	Time.

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Appendix B

BOUNDARY LAYER EQUATIONS

The general equations for viscous, constant-property fluid flow are quoted below. The Navier-Stokes equations of momentum and continuity in rectangular coordinates are:

Momentum:

$$\begin{aligned}\rho \frac{DU}{Dt} &= -\frac{\partial P}{\partial x} + \rho X + \mu \left[\nabla^2 U + \frac{1}{3} \frac{\partial}{\partial x} (\nabla \cdot \vec{V}) \right] \\ \rho \frac{DV}{Dt} &= -\frac{\partial P}{\partial y} + \rho Y + \mu \left[\nabla^2 V + \frac{1}{3} \frac{\partial}{\partial y} (\nabla \cdot \vec{V}) \right] \\ \rho \frac{DW}{Dt} &= -\frac{\partial P}{\partial z} + \rho Z + \mu \left[\nabla^2 W + \frac{1}{3} \frac{\partial}{\partial z} (\nabla \cdot \vec{V}) \right]\end{aligned}\quad (8)$$

Continuity:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho U) + \frac{\partial}{\partial y} (\rho V) + \frac{\partial}{\partial z} (\rho W) = 0 \quad (9)$$

The energy equation is:

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + W_i + W_f = \rho \frac{Dh}{Dt} - \frac{DP}{Dt} \quad (10)$$

where W_i is the rate at which work is done on a particle of fluid by an external field and W_f is the rate at which work is done against viscous stresses. Also, D/Dt represents the substantial derivative defined as

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$$

The Navier-Stokes equations are simplified by assuming two-dimensional, steady, incompressible flow in the absence of body forces for an ideal fluid of constant viscosity and thermal conductivity. Prandtl's order-of-magnitude treatment further reduces them until the result is:

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \frac{\partial^2 U}{\partial y^2} \quad (11)$$

APPENDIX A

CONTINUITY EQUATION

The general equation for mass, momentum, and energy balance in a control volume is given by the following equations of continuity and momentum in vectorial notation:

Continuity:

$$\rho \frac{D\rho}{Dt} = - \frac{\partial \rho}{\partial t} - \nabla \cdot (\rho \mathbf{V})$$

(1)

$$\rho \frac{D\mathbf{V}}{Dt} = - \frac{\partial \rho \mathbf{V}}{\partial t} + \nabla \cdot (\rho \mathbf{V} \mathbf{V})$$

$$\rho \frac{D\mathbf{W}}{Dt} = - \frac{\partial \rho \mathbf{W}}{\partial t} + \nabla \cdot (\rho \mathbf{W} \mathbf{V})$$

Continuity:

(2)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

The continuity equation is:

(3)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

where ρ is the mass density, \mathbf{V} is the velocity vector, and \mathbf{W} is the vorticity vector. The continuity equation is a statement of mass conservation in a fluid element.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

The Navier-Stokes equations are simplified by assuming two-dimensional, steady, incompressible flow in the absence of body forces. The resulting equations of motion are:

(4)

$$\rho \frac{D\mathbf{V}}{Dt} = - \nabla p + \mu \nabla^2 \mathbf{V}$$

and

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \quad (12)$$

for application to the boundary layer of a flat plate.

Similarly, the energy equation can be written in a simpler form as

$$U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \left(\frac{\partial U}{\partial x} \right)^2 \quad (13)$$

The last term, $\frac{\mu}{\rho c_p} \left(\frac{\partial U}{\partial x} \right)^2$, is usually considered negligible.

Thus the three equations of importance are:

Momentum:

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = - \frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \frac{\partial^2 U}{\partial y^2} \quad (14)$$

Continuity:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \quad (15)$$

Energy:

$$U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (16)$$

with boundary conditions

$$(a) \text{ at } y=0 : T=T_s \text{ or } \frac{\partial T}{\partial y} = - \frac{q}{kA} \text{ and } U=0 \quad (17)$$

$$(b) \text{ at } y=\infty : T=T_\infty \text{ and } U=U_\infty$$

Multiplying the continuity equation by U , adding the result to the momentum equation; and multiplying the continuity equation by T , adding the result to the energy equation, then simplifying both gives

$$\frac{\partial}{\partial x} (P + \rho U^2) = \frac{\partial}{\partial y} \left(\mu \frac{\partial U}{\partial y} - \rho UV \right) \quad (18)$$

$$\frac{\partial}{\partial x} (UT) = \frac{\partial}{\partial y} \left(\alpha \frac{\partial T}{\partial y} - VT \right) \quad (19)$$

and

(11)

$$0 = \frac{V_6}{\gamma_6} + \frac{U_6}{x_6}$$

for application to the boundary layer of a thin layer.

Similarly, the energy equation can be written in a similar form as

(12)

$$\left(\frac{U_6}{x_6}\right) \frac{1}{\gamma_6} + \frac{T_6}{\gamma_6} \frac{1}{\gamma_6} = \frac{T_6}{\gamma_6} V + \frac{T_6}{\gamma_6} U$$

The last term, $\left(\frac{U_6}{x_6}\right) \frac{1}{\gamma_6}$, is usually neglected negligible.

Thus the energy equation is important and

Simplification

(13)

$$\frac{U_6}{x_6} + \frac{T_6}{\gamma_6} \frac{1}{\gamma_6} = \frac{U_6}{x_6} V + \frac{U_6}{x_6} U$$

Continuity:

(14)

$$0 = \frac{V_6}{\gamma_6} + \frac{U_6}{x_6}$$

Energy:

(15)

$$\frac{T_6}{\gamma_6} \frac{1}{\gamma_6} = \frac{T_6}{\gamma_6} V + \frac{T_6}{\gamma_6} U$$

with boundary conditions

(16)

$$0 = U \text{ and } \frac{T}{K} = \frac{T_6}{\gamma_6} \text{ or } T = T_6 : 0 = \gamma \text{ to } (a)$$

$$\infty U = U \text{ and } T = T_6 : \infty = \gamma \text{ to } (b)$$

Multiplying the continuity equation by γ , adding the result to themomentum equation and multiplying the continuity equation by T , adding

the result to the energy equation, then simplifying gives

(17)

$$\left(\frac{U_6}{x_6} - \frac{U_6}{\gamma_6} \frac{1}{\gamma_6}\right) \frac{1}{\gamma_6} = \left(\frac{U_6}{x_6} + \frac{T_6}{\gamma_6}\right) \frac{1}{x_6}$$

(18)

$$\left(T_6 - \frac{T_6}{\gamma_6} \frac{1}{\gamma_6}\right) \frac{1}{\gamma_6} = \left(T_6 + \frac{T_6}{\gamma_6}\right) \frac{1}{x_6}$$

In turbulent flow Reynolds assumed that all quantities are made up of an average value plus a superposed fluctuation component such that

$$\begin{aligned} U &= \bar{U} + u \\ V &= \bar{V} + v \\ T &= \bar{T} + t \\ P &= \bar{P} + p \end{aligned} \quad (20)$$

Substituting these in equation (18) and (19), multiplying out and taking time averages:

$$\frac{\partial}{\partial x} (\bar{P} + \rho \bar{U}^2 + \rho \bar{u}^2) = \frac{\partial}{\partial y} \left(\mu \frac{\partial \bar{U}}{\partial y} - \rho \bar{U} \bar{V} - \rho \bar{uv} \right) \quad (1)$$

$$\frac{\partial}{\partial x} (\bar{U} \bar{T} + \bar{ut}) = \frac{\partial}{\partial y} \left(\alpha \frac{\partial \bar{T}}{\partial y} - \bar{V} \bar{T} - \bar{vt} \right) \quad (2)$$

In general \bar{uv} and \bar{vt} may be expected to have values other than zero as argued below: The \bar{U} velocity profile in a boundary layer over a flat plate is as shown in the sketch, Fig. 5. Choosing a point at x, y : If a 'packet' of air goes by the point in a $+V$ direction, air from closer to the plate will be forced into the 'hole' left by the packet. On the average, the entering air will have a U -velocity less than that which is average for the point x, y . This corresponds to a $(-u)$ as observed at that point.

Conversely, movement of a packet in the $(-v)$ direction gives rise to a $(+u)$ velocity.

Consequently, where there is a gradient of \bar{U} , a $(+v)$ produces a $(-u)$ and a $(-v)$ produces a $(+u)$, therefore the product is negative on the average. u and v are said to be 'cross-correlated' such that

$$\bar{uv} = \lim_{\tau \rightarrow \infty} \frac{1}{2\tau} \int_{-\tau}^{+\tau} (uv) d\tau \quad (23)$$

in Cartesian flow geometry, the following are used:

of an average value plus a superimposed fluctuation component such that

$$u = \bar{u} + u'$$

$$v = \bar{v} + v'$$

(10)

$$T = \bar{T} + T'$$

$$p = \bar{p} + p'$$

Substituting these in equation (1) and (2), multiplying out and taking

time averages:

$$(1) \quad \left(\bar{u} \bar{v} - \bar{u}' \bar{v}' - \frac{\bar{u} \bar{v}}{\gamma^2} \right) \frac{\gamma}{\gamma^2} = \left(\bar{u} \bar{v} + \bar{u}' \bar{v}' + \bar{u}' \bar{v}' \right) \frac{\gamma}{\gamma^2}$$

$$(2) \quad \left(\bar{T} \bar{v} - \bar{T}' \bar{v}' - \frac{\bar{T} \bar{v}}{\gamma^2} \right) \frac{\gamma}{\gamma^2} = \left(\bar{T} \bar{v} + \bar{T}' \bar{v}' + \bar{T}' \bar{v}' \right) \frac{\gamma}{\gamma^2}$$

In general $\bar{u}' \bar{v}'$ and $\bar{T}' \bar{v}'$ may be expected to have values other than zero

as argued below. The $\bar{u}' \bar{v}'$ velocity profile in a boundary layer over a flat

plate is as shown in the sketch, Fig. 2. The $\bar{u}' \bar{v}'$ profile is a

'pocket' of air near the wall in a + y direction, air from closer to

the plate will be forced into the 'pocket' by the pocket. As the flow

age, the pocket will have a $\bar{u}' \bar{v}'$ velocity less than zero which is shown

also for the case x, y . This corresponds to a $(-u')$ as observed in this

paper.

Consequently, instead of a pocket in the $(-u)$ direction there will

be a $(+u)$ velocity.

Consequently, where there is a gradient of \bar{u} , a $(+u)$ component is

$(-u)$ and a $(-v)$ component is $(+u)$. However the product is negative in

the average. u and v are said to be 'cross-correlated' when this

$$(11) \quad \overline{uv} = \lim_{\gamma \rightarrow \infty} \frac{1}{\gamma} \int_{-\gamma}^{+\gamma} (uv) dy$$

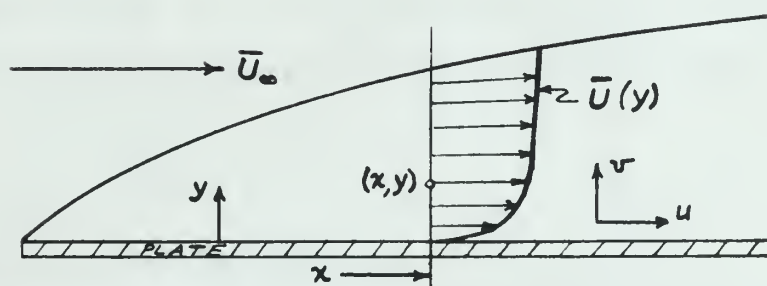


Fig. 5. Velocity Profile

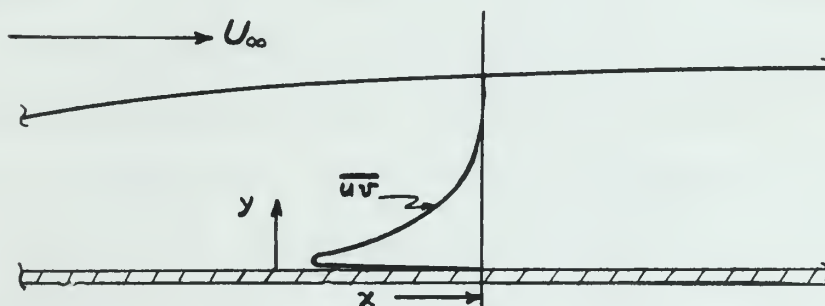


Fig. 6. Velocity Correlation Profile

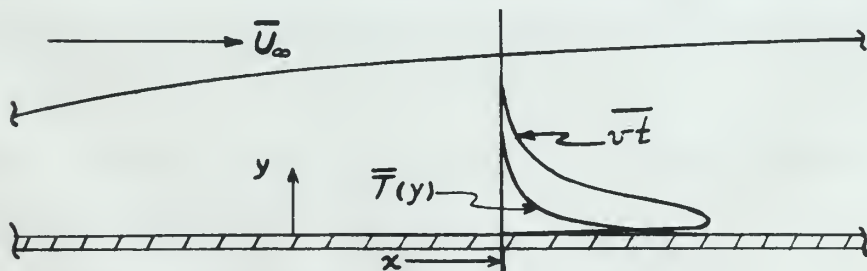


Fig. 7. Temperature and Velocity-Temperature Correlation Profiles

Close to the plate the fluid becomes laminar in the sub-layer, so u and v tend to zero. At the edge of the boundary layer where the gradient of U disappears, u and v tend to cross-correlate in a random fashion so the average is zero.

In summary, the expected behavior of \overline{uv} as a function of y should appear as shown in Fig. 6. Parallel reasoning leads one to expect a positive correlation of \overline{vt} where there is a gradient of \overline{T} such that

$$\overline{vt} = \lim_{\tau \rightarrow \infty} \frac{1}{2\tau} \int_{-\tau}^{+\tau} (vt) d\tau \quad (24)$$

These results are sketched in Fig. 7.

There is also a gradient of temperature in the x -direction producing a \overline{ut} correlation, but usually the gradient of \overline{T} in the x -direction is much smaller than in the y -direction, fluctuations in T are smaller, so \overline{ut} is expected to be an order of magnitude smaller than \overline{vt} . Johnson in reference (1) does show results, however, that indicate \overline{ut} to be about twice the magnitude of \overline{vt} . In this thesis \overline{ut} cancels out of the formulas used to calculate \overline{vt} . Its order of magnitude compared to \overline{vt} is therefore not determined.

It must be remembered that near the 'edge' of the thermal boundary layer \overline{ut} and \overline{vt} could become comparable; however the methods of measurement are unlikely to detect these quantities with much certainty near the free stream.

The concepts of eddy shear stress coefficient and eddy energy coefficient are defined by:

$$\overline{uv} = - \epsilon_m \frac{\partial \overline{U}}{\partial y} \quad (3)$$

and
$$\overline{vt} = - \epsilon_h \frac{\partial \overline{T}}{\partial y}$$

Class for the given \bar{y} (with boundary values in the sub-layer, no \bar{y} and a trend to zero. At the edge of the boundary layer where the gradient of \bar{y} disappears, a new \bar{y} level is introduced in a random fashion so the average is zero.

In summary, the expected behavior of \bar{y} as a function of \bar{y} should appear as shown in Fig. 6. The initial temperature levels are to expect a positive correlation at \bar{y} where there is a gradient of \bar{y} over time.

$$(20) \quad \bar{y} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} (y(t)) dt$$

These results are obtained in Fig. 7.

There is also a gradient of temperature in the x-direction produced a \bar{y} correlation, but usually the gradient of \bar{y} in the x-direction is much smaller than in the y-direction. The correlation for \bar{y} is smaller, so \bar{y} is expected to be an order of magnitude smaller than \bar{y} . However, in reference (1) it was shown that, however, that indicates it to be about twice the magnitude of \bar{y} . In this case we cannot say that the results used to calculate \bar{y} . The order of magnitude expected to be in this case and obtained.

It must be recognized that most of the 'edge' of the thermal boundary layer at \bar{y} would become negligible; however, the values of \bar{y} would not be helpful in determining correlation with \bar{y} and \bar{y} over the time interval.

The correlation of \bar{y} with \bar{y} is not sufficient to help easily understand

Related to (20)

$$(2) \quad \bar{y} = -\bar{y} \quad \bar{y} = -\bar{y}$$

(4)

where \mathcal{E}_m and \mathcal{E}_h are probably different functions of y .

When these definitions are substituted in equations (1) and (2), the result is

$$\frac{\partial}{\partial x} (\bar{P} + \rho \bar{U}^2 + \rho \bar{u}^2) = \frac{\partial}{\partial y} \left[(\mu + \rho \mathcal{E}_m) \frac{\partial \bar{U}}{\partial y} - \bar{U} \bar{V} \right] \quad (25)$$

and

$$\frac{\partial}{\partial x} (\bar{U} \bar{T} + \bar{u} \bar{t}) = \frac{\partial}{\partial y} \left[(\mathcal{L} + \mathcal{E}_h) \frac{\partial \bar{T}}{\partial y} - \bar{V} \bar{T} \right] \quad (26)$$

Solutions to these equations have been obtained for various simple geometries and assumed empirical distributions of pressure, velocity and temperature. In most cases the ratio of \mathcal{E}_m to \mathcal{E}_h is assumed to be constant or some arbitrary function of y . Very few measurements of \mathcal{E}_m and \mathcal{E}_h are available.

Appendix C*

HOT WIRE ANEMOMETER THEORY

A. Steady Flow.

Hot-wire theory has been well developed for various ideal cases. Most of the theory is based on King's equation which appears in various forms such as

$$\frac{H}{k(T_w - T_e)} = C_3 + C_4 \sqrt{N_{Pr} N_{Re}} \quad (27)$$

or alternatively

$$H = (T_w - T_e) \left[C_3 k + C_4 \sqrt{N_{Pr}} \sqrt{\frac{k^2 \rho}{\mu}} \sqrt{v_p D} \right] \quad (28)$$

For a gas, ρ is proportional to P/T_g , and for air $k^2/\mu T_g$ is approximately independent of pressure and temperature such that

$$H = (T_w - T_e) F(PV_p) \quad (29)$$

where

$$F(PV_p) = \left[0.58 + 6.6 \sqrt{P^* v_p^* D^*} \right] \quad (30)$$

The Flow Corporation, Cambridge, Massachusetts supplies hot-wire anemometer equipment which makes use of several interesting techniques. First, the equipment is of the 'constant current' type. Second, all measurements are made with the hot wire at a selected resistance ratio above the cold wire resistance. Third, measurements of unknown signals from the wire are made relative to a known square-wave signal applied to the wire from an internal circuit.

A summary of the principles of wire theory will give adequate under-

* This appendix is based primarily on references (3) and (12).

APPENDIX II

THEORY OF THE METHOD

A. THEORY

The theory of the method is based on the fact that the electrical resistance of a material is a function of its temperature. This is expressed by the following equation:

$$R = R_0 [1 + \alpha (T - T_0)] \quad (1)$$

where

$$R = \text{resistance at temperature } T$$
$$R_0 = \text{resistance at temperature } T_0$$
$$\alpha = \text{temperature coefficient of resistance}$$
$$T = \text{temperature in } ^\circ\text{C}$$
$$T_0 = \text{reference temperature in } ^\circ\text{C}$$

The resistance of a material is a function of its temperature. This is expressed by the following equation:

$$R = R_0 [1 + \alpha (T - T_0)] \quad (2)$$

$$R = R_0 [1 + \alpha (T - T_0)] \quad (3)$$

The first part of the theory is based on the fact that the electrical resistance of a material is a function of its temperature. This is expressed by the following equation:

The second part of the theory is based on the fact that the electrical resistance of a material is a function of its temperature. This is expressed by the following equation:

The third part of the theory is based on the fact that the electrical resistance of a material is a function of its temperature. This is expressed by the following equation:

A summary of the principles of the theory will give a more complete understanding of the method. This is expressed by the following equation:

standing of wire operation.

For a hypothetical fluctuating flow, the power input to the wire balances the power carried away by the stream,

$$I^2 R = H L = (T_w - T_e) L F(PV_p) \quad (31)$$

Also, neglecting second order terms,

$$R = R_0 \left[1 + \alpha_0 (T_w - T_0) \right] \quad (32)$$

where R_0 , α_0 , and T_0 are arbitrary corresponding quantities. These can be chosen as R_e , α_e and T_e at the 'cold' temperature of the wire assumed to be in equilibrium with the stream ambient temperature. Therefore

$$R = R_e \left[1 + \alpha_e (T_w - T_e) \right] \quad (33)$$

Solving (33) for $(T_w - T_e)$ and making use of $\mathcal{F} = R/R_e$, equation (31) becomes

$$I^2 \left(1 - \frac{1}{\mathcal{F}} \right) = \frac{L F(PV_p)}{\alpha_e R_e} \quad (34)$$

In steady state, therefore, a measurement of I will give the PV_p product since all other quantities are fixed. In practice for a particular wire at low velocities, a calibration of I vs V_p is made assuming F is nearly constant. Because it usually results in a straight line, the calibration is usually plotted as I^2 vs $\sqrt{V_p}$ giving ^a graph that may be represented by an equation of the form

$$I^2 = C_1 + C_2 \sqrt{V_p} \quad (35)$$

The derivative with respect to velocity is

$$\frac{dI}{dV_p} = \frac{C_2}{4 I \sqrt{V_p}} \quad (36)$$

In Flow Corporation literature, the quantity dI/dV_p is denoted by I' .

assuming all size operations.

For a hypothetical fluctuating flow, the power input to the wire would be given by the equation

$$(21) \quad I^2 R = \pi r^2 \int_0^L (T_w - T_0) dx$$

Also, neglecting second order terms,

$$(22) \quad I^2 R = \pi r^2 \left[L(T_w - T_0) + \frac{1}{2} \alpha L^2 (T_w - T_0)^2 \right]$$

where T_w , T_0 , and α are the average corresponding quantities. There can be errors as T_w and T_0 are the 'real' temperatures of the wire exposed to be in equilibrium with the stream which is important. Therefore

$$(23) \quad I^2 R = \pi r^2 \left[L(T_w - T_0) + \frac{1}{2} \alpha L^2 (T_w - T_0)^2 \right]$$

Solving (23) for $(T_w - T_0)$ and using one of $\alpha = 1/100$, equation

(24) becomes

$$(24) \quad T_w - T_0 = \frac{I^2 R}{\pi r^2 L} \left(1 - \frac{1}{2} \alpha L^2 \right)$$

In steady state, therefore, a measurement of I will give the T_w provided that all other quantities are fixed. In practice for a particle that moves at low velocities, a calibration of I vs T_w is made assuming I is nearly constant. Because it usually results in a straight line, the calibration is usually plotted as I^2 vs $\sqrt{V_p}$. Using graphs that may be represented by an equation of the form

$$(25) \quad I^2 = C_1 + C_2 \sqrt{V_p}$$

The derivative with respect to velocity is

$$(26) \quad \frac{dI^2}{d\sqrt{V_p}} = \frac{C_2}{2\sqrt{V_p}}$$

In this hypothetical literature, the density ρ is assumed to be 1 .

3. Turbulent Flow.

When the stream is turbulent, no steady state exists and the operation of the wire must be analyzed from a time dependent approach. The wire stores heat. Since it cannot change heat storage instantaneously, a power loss balance for the wire is

$$c A L \frac{\partial T_w}{\partial \tau} = I^2 R - \frac{L (\mathcal{F} - 1)}{\mathcal{L}_e} F(PV_p) \quad (37)$$

From equation (33)

$$\frac{\partial T_w}{\partial \tau} = \frac{1}{\mathcal{L}_e R_e} \frac{\partial R}{\partial \tau} \quad (38)$$

$$\text{Take } R = \bar{R} + r \quad (39)$$

$$\text{and } F(PV_p) = F(\bar{P}\bar{V}_p + \bar{P}v_p) \quad (40)$$

where also

$$F(\bar{P}\bar{V}_p + \bar{P}v_p) = F(\bar{P}\bar{V}_p) + \left. \frac{\partial F}{\partial v_p} \right|_p v_p = \bar{F}(PV_p) + F' v_p \quad (41)$$

Substituting these in equation (37), expanding and subtracting the steady state equation (31) leaves

$$c A L \frac{\partial r}{\partial \tau} = \mathcal{L}_e R_e I^2 r - L (\bar{R} - R_e) F' v_p - L \bar{F} r \quad (42)$$

Since the voltage across the wire is

$$e_w = I r \quad (43)$$

equation (42) becomes

$$c A L \frac{d e_w}{d \tau} = [\mathcal{L}_e R_e I^2 - L \bar{F}] e_w - L (\bar{R} - R_e) I F' v_p \quad (44)$$

Solving for v_p :

$$v_p = - \left[\frac{L \bar{F} - \mathcal{L}_e R_e I^2}{L (\bar{R} - R_e) I F'} e_w + \frac{c A L}{L (\bar{R} - R_e) I F'} \frac{d e_w}{d \tau} \right] \quad (45)$$

Thus, the fluctuating velocity component, v_p , is a function of the voltage and the time derivative of the voltage induced in the wire by the velocity. The Flow Corporation HWS performs this operation such that the

8. Further from

when the system is perturbed, we already know the value and the operation of the system must be analyzed from a time dependent approach. The also known fact. Since it cannot change with respect to time, a general form follows for the rate as

$$(37) \quad \frac{dV}{dt} = \frac{1}{C} \left(\frac{1}{R} - \frac{1}{R_0} \right) \quad \text{From equation (33)}$$

$$(38) \quad \frac{dV}{dt} = \frac{1}{C} \left(\frac{1}{R} - \frac{1}{R_0} \right)$$

$$(39) \quad \frac{dV}{dt} = \frac{1}{C} \left(\frac{1}{R} - \frac{1}{R_0} \right)$$

$$(40) \quad \frac{dV}{dt} = \frac{1}{C} \left(\frac{1}{R} - \frac{1}{R_0} \right)$$

$$(41) \quad \frac{dV}{dt} = \frac{1}{C} \left(\frac{1}{R} - \frac{1}{R_0} \right)$$

$$(42) \quad \frac{dV}{dt} = \frac{1}{C} \left(\frac{1}{R} - \frac{1}{R_0} \right)$$

$$(43) \quad \frac{dV}{dt} = \frac{1}{C} \left(\frac{1}{R} - \frac{1}{R_0} \right)$$

$$(44) \quad \frac{dV}{dt} = \frac{1}{C} \left(\frac{1}{R} - \frac{1}{R_0} \right)$$

$$(45) \quad \frac{dV}{dt} = \frac{1}{C} \left(\frac{1}{R} - \frac{1}{R_0} \right)$$

$$(46) \quad \frac{dV}{dt} = \frac{1}{C} \left(\frac{1}{R} - \frac{1}{R_0} \right)$$

$$(47) \quad \frac{dV}{dt} = \frac{1}{C} \left(\frac{1}{R} - \frac{1}{R_0} \right)$$

$$(48) \quad \frac{dV}{dt} = \frac{1}{C} \left(\frac{1}{R} - \frac{1}{R_0} \right)$$

$$(49) \quad \frac{dV}{dt} = \frac{1}{C} \left(\frac{1}{R} - \frac{1}{R_0} \right)$$

Thus, the fluctuating velocity component, v' , is a function of the voltage and the time derivative of the voltage induced in the wire by the voltage. The time derivative of the voltage induced in the wire by the voltage is

output voltage is

$$e_s = J_1 e_w + J_2 \frac{d e_w}{d \tau} \quad (46)$$

J_2 is a constant but J_1 is adjustable so that

$$J_1 = J_2 \left(\frac{L \bar{F} - \mathcal{L}_e R_e I^2}{c A L} \right) \quad (47)$$

Solving (45) for e_w or $d e_w / d \tau$ and substituting in (46)

$$e_s = - \left[\frac{J_1 L (\bar{R} - R_e) I F'}{L \bar{F} - \mathcal{L}_e R_e I^2} \right] v_p = - \left[\frac{J_2 I (\bar{R} - R_e) F'}{c A} \right] v_p \quad (48)$$

if equation (47) is true. The adjustment of the ratio of J_1 / J_2 is made by observing the amplifier output on an oscilloscope when a square-wave current is fed to the wire. The process of 'compensating' the amplifier is detailed in Appendix D. At the point of proper compensation with the wire in still air, the wire response to the square-wave current is

$$e_c = \left[\frac{2 J_1 \bar{I}^2 \bar{R} R_e \mathcal{L}_e}{L \bar{F} - \bar{I}^2 \mathcal{L}_e R_e} \right] i_s = \left[\frac{2 J_2 \bar{I}^2 \bar{R} R_e \mathcal{L}_e}{c A L} \right] i_s \quad (49)$$

$$\text{where } I = \bar{I} + i_s \quad (50)$$

The ratio of e_s to e_c is

$$\frac{e_s}{e_c} = \left[\frac{L (\bar{R} - R_e) I F'}{2 \bar{I}^2 \bar{R} R_e \mathcal{L}_e i_s} \right] v_p \quad (51)$$

$$\text{Since } F' = \frac{2 \mathcal{L}_e R_e}{L} \left(\frac{\mathcal{F}}{\mathcal{F}-1} \right) \bar{I} I' \quad (52)$$

$$\text{then } \frac{e_s}{e_c} = \frac{I'}{i_s} v_p, \text{ or } v_p = \frac{i_s}{I'} \frac{e_s}{e_c} \quad (53)$$

(16)

$$\frac{1}{\gamma} \left(\frac{1}{\gamma} + \frac{1}{\gamma} \right) = 1$$

(17)

$$\frac{1}{\gamma} \left(\frac{1}{\gamma} + \frac{1}{\gamma} \right) = 1$$

Solving (16) for γ or γ and substituting in (17)

$$(18) \quad \left[\frac{1}{\gamma} \left(\frac{1}{\gamma} + \frac{1}{\gamma} \right) \right] = 1$$

is equation (18) is given. The adjustment of the ratio γ is well

by observing the behavior of the coefficient of the ratio γ in equation

is given in equation (18). The process of 'normalizing' the coefficient is

described in equation (18). At the point of interest, the coefficient is

in all the cases, the response to the input is given by

$$(19) \quad \left[\frac{1}{\gamma} \left(\frac{1}{\gamma} + \frac{1}{\gamma} \right) \right] = 1$$

(20)

$$\frac{1}{\gamma} + \frac{1}{\gamma} = 1$$

The ratio of γ to γ is

(21)

$$\left[\frac{1}{\gamma} \left(\frac{1}{\gamma} + \frac{1}{\gamma} \right) \right] = 1$$

(22)

$$\frac{1}{\gamma} + \frac{1}{\gamma} = 1$$

(23)

$$\frac{1}{\gamma} + \frac{1}{\gamma} = 1$$

It must be realized that in the derivation of all equations, fluctuating components are small compared with the average values. This allows neglecting second-order products of the fluctuating components as being immeasurably small. Even when this is not strictly true, the method of normalizing represented by equation (51) compensates for some deviation.

C. Turbulent Flow with Temperature Fluctuations.

A fluid in turbulent flow past a heated surface will become heated in some fluctuating manner. The effect of varying fluid temperature may be regarded as a change in the 'cold' resistance of the wire, R_c , such that

$$R_g = \bar{R}_g + r_g \quad (54)$$

$$= R_c \left[1 + \alpha_e (T_g - T_a) \right] \quad (55)$$

Combining these with the power balance equations in the presence of velocity fluctuations, plus the relations

$$\begin{aligned} e_w &= I r \\ e_s &= J_1 \left(e_w + \frac{c A L}{L \bar{F} - \bar{I}^2 \alpha_e R_c} \frac{d e_w}{d z} \right) \\ e_c &= \left(\frac{2 J_1 \bar{I}^2 \bar{R} R_c \alpha_e}{L \bar{F} - \bar{I}^2 R_c \alpha_e} \right) i_s \\ F' &= \frac{2 \bar{I} I' \alpha_e R_c}{L} \frac{\bar{R}}{\bar{R} - \bar{R}_g}, \quad \text{and} \\ F(PV_p) &= \frac{\alpha_e R_c \bar{I}^2 \bar{R}}{L (\bar{R} - \bar{R}_g)} \end{aligned}$$

gives

$$2 i_s \frac{e_s}{e_c} + 2 I' v_p - \frac{\bar{I}}{\bar{R} - \bar{R}_g} r_g = 0 \quad (56)$$

It must be pointed out in the derivation of all equations, the
 existing components are still regarded with the average values. This al-
 lows regarding known-order functions of the fluctuating component as
 being statistically small. Now, when this is not strictly true, the
 method of perturbation representation by equation (11) requires the use
 of iteration.

7. Perturbations with Temperature Fluctuations

A field in equilibrium with a heated system will become heated
 in some characteristic manner. The effect of varying field components may
 be regarded as a change in the 'cold' resistance of the wire, R_{00} , such

$$(20) \quad R = R_0 + \delta R$$

$$(21) \quad \left[\frac{1}{R} - \frac{1}{R_0} \right] = \frac{\delta R}{R_0^2}$$

Combined these with the power balance equation in the presence of

velocity fluctuations, from the relations

$$\frac{1}{R} = \frac{1}{R_0} + \frac{\delta R}{R_0^2}$$

$$\frac{1}{R} = \frac{1}{R_0} + \frac{\delta R}{R_0^2}$$

$$\frac{1}{R} = \frac{1}{R_0} + \frac{\delta R}{R_0^2}$$

$$\frac{1}{R} = \frac{1}{R_0} + \frac{\delta R}{R_0^2}$$

$$(22) \quad \frac{1}{R} = \frac{1}{R_0} + \frac{\delta R}{R_0^2}$$

Noting that $T_g = \bar{T}_g + t$ (57)

$$\bar{R}_g = R_e \left[1 + \alpha_e (T_g - T_e) \right] \quad (55)$$

and

$$r_g = \alpha_e R_e t \quad (58)$$

leads to

$$2 I_s \frac{e_s}{e_c} + 2 I' v_p - \frac{\alpha_e \bar{I}}{\gamma - [1 + \alpha_e (\bar{T}_g - T_e)]} t = 0 \quad (59)$$

e_s and e_c have been taken as the outputs from an ideal amplifier.

An actual amplifier will have an output of $(e_s + e_n)$ and $(e_c + e_n)$. When this output is fed to a true RMS voltmeter, the latter will have a reading of

$$M_{sn} = K_1 \sqrt{\lim_{\tau \rightarrow \infty} \frac{1}{2\tau} \int_{-\tau}^{+\tau} (e_s + e_n)^2 d\tau} \quad (60)$$

$$\text{or } M_{cn} = K_1 \sqrt{\lim_{\tau \rightarrow \infty} \frac{1}{2\tau} \int_{-\tau}^{+\tau} (e_c + e_n)^2 d\tau} \quad (61)$$

But the time average of the voltage is $\overline{(e_s + e_n)^2} = \overline{e_s^2} + \overline{e_n^2}$

since the time average of $\overline{e_s e_n} = 0$. That is, there is no correlation between noise and signal.

Therefore,

$$M_{sn} = K_1 \sqrt{\overline{e_s^2} + \overline{e_n^2}} \quad (62)$$

and

$$M_n = K_1 \sqrt{\overline{e_n^2}} \quad (63)$$

$$\text{It follows that } \sqrt{M_{sn}^2 - M_n^2} = K_1 \sqrt{\overline{e_s^2}} \quad (64)$$

the RMS value of e_s . For the square-wave

$$\sqrt{M_{scn}^2 - M_{sn}^2} = K_1 \sqrt{\overline{e_c^2}} = K_1 e_c. \quad (65)$$

Squaring equation (59) and making use of the meter relations gives

$$(27) \quad \bar{z} + \bar{z} = \bar{z} \quad \text{writing time}$$

$$(28) \quad \left[(z - \bar{z})^2 + 1 \right] z = \bar{z}$$

$$(29) \quad z + \bar{z} = z$$

$$(30) \quad z = \frac{\bar{z}^2}{\left[(z - \bar{z})^2 + 1 \right]} + \frac{z}{z} + \frac{z}{z} = z$$

and z have been taken as the output from the input amplifier. The second amplifier will have an output of $(z + \bar{z})$ and $(z + \bar{z})$ and this output is fed to a summing junction, the output will have a reading of

$$(31) \quad \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^{+T} (z + \bar{z})^2 dt$$

$$(32) \quad \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^{+T} (z + \bar{z})^2 dt = \frac{z}{z} + \frac{\bar{z}}{\bar{z}} = z + \bar{z}$$

but the time average of the voltage is $\frac{z}{z} + \frac{\bar{z}}{\bar{z}} = z + \bar{z}$ since the time average of $z = \bar{z}$. Thus the output is the same as the input.

$$(33) \quad \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^{+T} (z + \bar{z})^2 dt = z + \bar{z}$$

$$(34) \quad \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^{+T} (z + \bar{z})^2 dt = z + \bar{z}$$

$$(35) \quad \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^{+T} (z + \bar{z})^2 dt = z + \bar{z}$$

the output of z . For the input

$$(36) \quad \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^{+T} (z + \bar{z})^2 dt = z + \bar{z}$$

integrating equation (36) and taking use of the output relation gives

$$\begin{aligned}
 (2 I_s)^2 \left[\frac{M_{scn}^2 - M_{si}^2}{M_{scn}^2 - M_{sn}^2} \right] &= (2 I')^2 \frac{v_p^2}{t^2} - \left\{ \frac{4 \alpha_e \bar{I} I'}{\sigma - [1 + \alpha_e (\bar{T}_g - T_e)]} \right\} \frac{v_p}{t} \\
 &+ \left\{ \frac{\alpha_e \bar{I}}{\sigma - [1 + \alpha_e (\bar{T}_g - T_e)]} \right\}^2 \frac{1}{t^2} \quad (66)
 \end{aligned}$$

A set of simultaneous equations can be obtained by measuring all quantities at several resistance ratios. For more than three resistance ratios the unknowns are overdetermined and the method of averages or least squares can be used to evaluate the unknowns.

D. Resolution of Velocity Components.

The foregoing analysis assumed that the velocity was perpendicular to the axis of the wire. Within 70° or more from the perpendicular, the cooling effect on the wire varies as the cosine of the angle, that is $V_\theta = V_p \cos \theta$. A wire at 45° to the average stream direction has an average effective velocity $1/\sqrt{2}$ times the stream velocity. This applies equally to the steady value and the fluctuating value of the velocity.

Consider only the fluctuating components in the turbulent stream, because the amplifier filters out all DC wire response components. For two wires at $\pm 45^\circ$ to the stream,

$$v_a = \frac{u + v}{\sqrt{2}} \quad (67)$$

$$v_b = \frac{u - v}{\sqrt{2}} \quad (68)$$

Setting $v_a = v_p$, then $v_b = v_p$, equation (66) becomes

$$(66) \quad \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left\{ \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right] \right\} + \left\{ \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right] \right\} = \left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right]^2$$

A set of simultaneous equations can be obtained by assuming all quantities as general variables rather than as specific values. The ratios the unknowns are determined and the system of equations of least squares can be used to evaluate the unknowns.

5. Evaluation of Velocity Components

The foregoing analysis assumes that the velocity was perpendicular to the axis of the wire. Figure 10 shows that the perpendicularity, the cooling effect on the wire varies as the cosine of the angle, that is, $\cos \theta = \frac{v}{c}$. If θ is the angle between the velocity and the axis of the wire, the average effective velocity is $\frac{1}{\sqrt{2}}$ times the actual velocity. This gives exactly the same value and the theoretical value of the velocity.

Consider only the fluctuating component in the horizontal stress, because the vertical stress is not all in the vertical component. The two wires are $\pm 45^\circ$ to the vertical.

$$(67) \quad \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$(68) \quad \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Because $v = c$, then $v = c$, equation (68) becomes

$$\begin{aligned}
(2 i_s)_a^2 \left[\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right]_a &= 2 I_a'^2 (\overline{u^2} + 2\overline{uv} + \overline{v^2}) \\
&\quad - \frac{2\sqrt{2} \mathcal{L}_e I_a' \bar{I}_a}{\gamma' - [1 + \mathcal{L}_e (\bar{T}_g - T_e)]} (\overline{ut} + \overline{vt}) \\
&\quad + \left\{ \frac{\mathcal{L}_e \bar{I}_a}{\gamma' - [1 + \mathcal{L}_e (\bar{T}_g - T_e)]} \right\}^2 \overline{t^2}
\end{aligned} \tag{69}$$

and

$$\begin{aligned}
(2 i_s)_b^2 \left[\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right]_b &= 2 I_b'^2 (\overline{u^2} - 2\overline{uv} + \overline{v^2}) \\
&\quad - \frac{2\sqrt{2} \mathcal{L}_e I_b' \bar{I}_b}{\gamma' - [1 + \mathcal{L}_e (\bar{T}_g - T_e)]} (\overline{ut} - \overline{vt}) \\
&\quad + \left\{ \frac{\mathcal{L}_e \bar{I}_b}{\gamma' - [1 + \mathcal{L}_e (\bar{T}_g - T_e)]} \right\}^2 \overline{t^2}
\end{aligned} \tag{70}$$

Subtracting (69) from (70):

$$\begin{aligned}
4 \left\{ i_{sb}^2 \left[\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right]_b - i_{sa}^2 \left[\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right]_a \right\} &= \\
2 (I_b'^2 - I_a'^2) (\overline{u^2} + \overline{v^2}) & \\
- 4 (I_b'^2 + I_a'^2) \overline{uv} & \\
- \frac{2\sqrt{2} \mathcal{L}_e}{\gamma' - [1 + \mathcal{L}_e (\bar{T}_g - T_e)]} (I_b' \bar{I}_b - I_a' \bar{I}_a) \overline{ut} & \\
+ \frac{2\sqrt{2} \mathcal{L}_e}{\gamma' - [1 + \mathcal{L}_e (\bar{T}_g - T_e)]} (I_b' \bar{I}_b + I_a' \bar{I}_a) \overline{vt} & \\
+ \left\{ \frac{\mathcal{L}_e}{\gamma' - [1 + \mathcal{L}_e (\bar{T}_g - T_e)]} \right\}^2 (\bar{I}_b^2 - \bar{I}_a^2) \overline{t^2} &
\end{aligned} \tag{71}$$

Fortunately, the two most important quantities of interest in the boundary layer analysis are the dominant terms in this equation. It reduces to

$$\begin{aligned}
 (2 \text{ } I_s)^2 \left[\left(\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right)_b - \left(\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right)_a \right] &= - 8 \text{ } I'^2 \overline{uv} \\
 &+ \frac{2\sqrt{2} (\bar{I}_a + \bar{I}_b) \text{ } I' \mathcal{L}_e}{\delta' - [1 + \mathcal{L}_e (\bar{T}_g - T_e)]} \overline{vt} \quad (72)
 \end{aligned}$$

for two ideally matched wires or a single wire turned successively to the two orientations of $\pm 45^\circ$ to the stream.

consequently, the two most important quantities of interest in the present study are the quantities α and β . In the case of

$$(2.1) \quad \frac{\partial}{\partial \alpha} \left[\frac{\left(\frac{\alpha}{\alpha^2 + \beta^2} - \frac{\beta}{\alpha^2 + \beta^2} \right) \left(\frac{\alpha}{\alpha^2 + \beta^2} - \frac{\beta}{\alpha^2 + \beta^2} \right)}{\left(\frac{\alpha}{\alpha^2 + \beta^2} - \frac{\beta}{\alpha^2 + \beta^2} \right)} \right] = -\frac{2\alpha\beta}{\alpha^2 + \beta^2}$$

for the two special cases where α and β are functions of the same variable, the two quantities of interest are $\pm \frac{1}{2}$ to the order of

APPENDIX D

EXPERIMENTAL EQUIPMENT AND FACILITIES

(a) Wind Tunnel

The experiments were conducted in a tunnel whose test area has a one foot square cross section. Atmospheric air was drawn through the tunnel by an exhaust fan powered by a one quarter horsepower AC motor. The flow rate through the test section was controlled by a damper in the exhaust line. The maximum velocity attained was approximately twenty-six feet per second. The velocities were measured by a pilot tube probe located on the axis of the tunnel three feet upstream from the test section. A micromanometer was connected to this probe.

The boundary layer was developed over a flat plate which consisted of a twenty-nine inch unheated starting length and a twelve inch heated section. The heating element provided approximately constant heat transfer per unit area. The blunt leading edge of the unheated plate tripped the momentum boundary layer. As shown in Appendix E, the thickness of the momentum and thermal boundary layers at the test station were estimated to be 1.32" and 0.48" respectively.

(b) True RMS Millivoltmeter

An instrument that measures the true root-mean-square voltage of a random input signal was required. There are two commercial meters available which perform this function. They are the Ballantine Laboratories, Inc., model 320 and Flow Corporation, model TM. The principal difference between them is the manner in which the input signal is averaged. The

model 320 uses an amplifier and a squaring circuit of the segmented type. The model TBM employs a unique design thermal element. The principal characteristics of these instruments are:

<u>Characteristic</u>	<u>Model 320</u>	<u>TBM</u>
meter range (rms volts)	0.0001-320	0.0001-250
frequency response (cycles per second)	15-150,000	2-250,000
linearity over frequency range	$\pm 3\%$	$\pm 2\%$
time constant, including meter movements (seconds)	2.25	15
peak factor $\frac{P}{E}$ ratio of period to pulse width	—	10
crest factor $\frac{P}{E}$ ratio of peak to rms amplitude	5	—

Due to the presence of many low frequency components in the turbulence velocities being measured, a relatively long averaging period is required for accurate results. The problems encountered with these meters are presented in Section IV.

(c) Probes

Both a single wire and a V-wire probe were used in this series of experiments. Each had tungsten wires of 0.00015" diameter with sensitive lengths of 0.040". These probes are shown in figure 4.

The single wire probe was used to simulate an X-wire probe as noted in the discussion of results. The best results were obtained in this manner. The V-wire probe had a 90° included angle, and the sensitive segments of the wires were located as close as possible to the apex. The

model 320 uses an oscillator and a recording circuit of the standard type. The model 320 employs a unique design circuit element. The following characteristics of these instruments are:

Model 320	Model 320	Model 320
0.0001-100	0.0001-100	0.0001-100
2-100,000	2-100,000	2-100,000
$\pm 1\%$	$\pm 1\%$	$\pm 1\%$
12	12	12
10	10	10
—	—	—

Due to the presence of water in the frequency components in the analysis velocity being measured, a relatively long averaging period is required for accurate results. The frequency components with these values are recorded in Section 17.

(c) Probes

Both a single wire and a V-wire probe were used in this section of experiments. Each had frequency range of 0.0001-100,000 cycles per second. These probes are shown in Figure 4.

The single wire probe was used to obtain an 8-10 cps response noted in the discussion of results. The best results were obtained in this manner. The V-wire probe had a 90° included angle, and the sensitivity of the wire probe located as close as possible to the apex. The

resistance of the wires was matched to less than two percent. The problems previously discussed severely limited the use of this probe.

The probes were mounted on a traversing mechanism with a micrometer adjustment to accurately measure the distance from the heated plate. A calibrated index was used to change the angular orientation of the probe.

A carefully constructed X-wire probe correctly oriented to the free stream velocity vector would satisfy the assumptions made in the derivation of equation (72). It would eliminate the principal problems encountered with the V-wire probe - i.e., space resolution and needle resistance. Such a probe was not constructed for this thesis due to fund limitations.

(d) Anemometer Circuits

There are three principal circuits required to transform the wire signal into the data required by equation (72). These are the resistance bridge, square-wave circuit, and amplifier. They are conveniently packaged in the Flow Corporation HNB unit. No other commercially available anemometer equipment meets the requirements for measuring fluctuation quantities.

The bridge circuit provides a method for accurately measuring the wire resistance. This is the fundamental quantity which is sensitive to velocity and temperature fluctuations. The resistance of the wire at any operating condition is calculated from measurements of current (\bar{I}), voltage (C_w) and resistance ratio (γ).

The amplifier section boosts the AC components of the voltage across the hot wire to a readable level. It also takes the time derivative of the wire voltage and adds it to a fraction of the wire voltage. This fraction is selected by adjusting the compensation dial to obtain minimum

testament of the wire was returned to less than the error. The problems previously discussed were limited to the use of this probe. The probe was mounted on a traveling mechanism with a constant adjustment to accurately measure the distance from the probe plane. A calibrated index was used to change the angular relationship of the probe. A carefully constructed frame was carefully aligned to the fixed stress velocity vector would verify the assumption made in the derivation of equation (12). It really eliminates the principal problem associated with the 9-wire probe - i.e., speed resolution and transfer resistance. This probe was not mounted on this frame due to time limitations.

(d) Intermittent Operation

There are three principal elements required to transmit the wire signal into the tape recorder by equation (12). These are the resistance bridge, square-wave circuit, and amplifier. They are conventionally available in the wire measurement and wire. In other commercially available measurement equipment such as the tape recorder for measuring fluctuation quantities.

The bridge circuit provides a means for accurately measuring the wire resistance. This is the fundamental quantity which is sensitive to velocity and temperature fluctuations. The resistance of the wire at any operating condition is calculated from measurement of current (1), voltage (2), and resistance ratio (3).

The amplifier section provides the AC component of the bridge across the AC wire to a readable level. It also takes the time derivative of the wire voltage and also it is a function of the wire voltage. This function is selected by adjusting the compensation also as shown in the

signal distortion. To properly set the compensation, a square wave is impressed across the bridge circuit and the amplified wire signal is displayed on an oscilloscope. Correct compensation is obtained when the square wave appears linear with a sharp peak at the leading edge. This vertical peak superimposed on a square wave represents the addition of the derivative to the input signal.

Any random input signal to the amplifier may be represented as a Fourier series. The output voltage is therefore a function of frequency. For accurate measurements all of the frequency components should be passed by the amplifier. The HWB has a quoted bandwidth of 2 cps to 100,000 cps. A further error would be introduced by overdriving the amplifier. To prevent this, the attenuator dial was set at '16' which corresponds to 1/16 of the maximum gain.

The square wave circuit provides a reference voltage to the wire for two purposes. First, the square wave is employed to set the amplifier compensation. In addition, it is used to calibrate or normalize the wire voltage that results from stream fluctuations.

One other piece of anemometer equipment, the HWI, was used to a limited extent. It contains a wire current control circuit and a sum-and-difference circuit. It is used in conjunction with the HWB unit and a dual-wire probe. The sum-and-difference of the instantaneous signals from two wires is required to calculate fluctuating velocity components.

(e) Oscilloscope

The output signal of the anemometer amplifier was displayed on an oscilloscope. A Dumont, type 322, dual channel instrument was available in the laboratory. The dual beam presentation will be found particularly useful with an X-wire probe.

signal detection. To prevent the possibility of a false alarm is
represented between the bridge circuit and the amplifier with a gain of
displayed on an oscilloscope. Current compensation is obtained from the
applies wave square wave with a duty cycle at the bridge side. This
vertical wave is superimposed on a square wave represents the position of
the derivative to the input signal.

Any random noise signal in the amplifier may be represented as a
Fourier series. The output voltage is therefore a function of frequency.
For accurate measurements all of the frequency components should be passed
by the amplifier. The BW has a power spectrum of 1 cps to 100,000 cps.
A further noise would be introduced by modulating the amplifier. In
general this, the amplifier will not be of 1% which represents 10
1/10 of the maximum gain.

The output noise circuit provides a reference voltage to the side the
the purpose. Thus, the output may be applied to the amplifier
connection. In addition, it is used to calibrate or normalize the data
voltage that provides good signal-to-noise ratio.

One other aspect of amplifier operation, the BW, was used to a
limited extent. It contains a wide variety of control circuits and a gain
and offset control circuit. It is used in conjunction with the BW and has
a dual-slope process. The pre-amplifier of the instrument is a
from the noise is removed so that the amplifier is only concerned.

(a) Amplifier

The output signal of the instrument amplifier was displayed on an
oscilloscope. A power, 100 W, dual channel instrument was available
in the laboratory. The dual beam operation will be found particularly
useful with an X-ray probe.

APPENDIX E

DATA CALCULATIONS

1. Wire Calibration

Before each wire was calibrated, it was "aged" at a heating current slightly above the maximum operating range. Its velocity response was determined by calibration in the free stream of the tunnel. The wire was oriented perpendicular to the flow and measurements were made of wire current for various resistance ratios. The air velocity was controlled by a damper in the tunnel exhaust line.

A pitot tube with a micromanometer was used to determine the air velocity. A light oil of 0.827 specific gravity filled the manometer tube. All calculations included corrections for atmospheric pressure and temperature in the tunnel; however, since the temperature variations were small, no correction was made for the density changes of the oil. The following equations were employed to calculate velocities in the tunnel:

$$\begin{aligned}\bar{U} &= \sqrt{2gh} = \left[2(g)(h) \frac{0.827 \rho_W}{\rho_A} \right]^{\frac{1}{2}} \\ &= \left[2(32.2 \text{ ft/sec}) \frac{h \text{ inches}}{12 \text{ inches/ft}} \frac{0.827 \times 62.2 \text{ lbs/ft}^3}{\rho_A \text{ lbs/ft}^3} \right]^{\frac{1}{2}} \\ \bar{U} &= 16.6 \left[\frac{h}{\rho_A} \right]^{\frac{1}{2}}\end{aligned}$$

$$\text{where, } \rho_A = \frac{P^{15^\circ\text{C}}_{760\text{mm}} (0.3789) (p \text{ mm Hg})}{(t^\circ\text{C} + 273.20)} = \frac{(0.07657)(0.3789)(p)}{(t + 273.2)}$$

$$\rho_A = \frac{0.02901 p}{(t + 273.2)}$$

DATA CALCULATION

1. Viscosity Calculation

Before each run was completed, it was noted as a viscosity reading slightly above the normal operating range. The viscosity readings were determined by rotation in the film between the plates. The air was released perpendicular to the flow and measurements were made of the current for various resistance values. The air velocity was controlled by a meter in the tunnel exhaust line.

A glass tube with a thermometer was used to measure the air velocity. A flow rate of 0.025 cubic feet per minute was maintained. All calculations required corrections for density, pressure and temperature in the tunnel. Density, since the temperature variations were small, no correction was made for the density change of the air. The following equation was employed to calculate viscosity in the

Results:

$$\bar{v} = \sqrt{\frac{2g\Delta h}{\rho}} = \left[\frac{2(9.81)(0.1117)}{\rho} \right]^{1/2}$$

$$= \left[\frac{2(9.81)(0.1117)}{\rho} \right]^{1/2} = \frac{0.025 \text{ (cubic ft/min)}}{0.0001 \text{ (sq ft)}} \times \frac{1}{60} \times \frac{1}{1.225 \text{ (kg/m}^3\text{)}}$$

$$\bar{v} = 10.6 \left[\frac{\mu}{\rho} \right]^{1/2}$$

$$\text{Reynolds } Re = \frac{\rho \bar{v} D}{\mu} = \frac{(1.225)(10.6) \left[\frac{\mu}{\rho} \right]^{1/2} (0.0001)}{\mu} = \frac{0.0001225 (10.6)^{1/2}}{\mu^{1/2}}$$

$$Re = \frac{0.0001225}{\mu^{1/2}} \times 10.6^{1/2}$$

Typical calculations for two resistance ratios with data taken on 7/3/58 follows:

$$P_A = 762.15 \text{ mm Hg} ; \quad t_A = 27.5^\circ \text{ C}$$

$$\rho_A = \frac{(0.02901)(762.15)}{(27.5 + 273.2)} = \frac{22.110}{300.7} = 0.07352 \text{ lbs/ft}^3$$

$$\bar{U} = 16.6 \left[\frac{h}{0.07352} \right]^{\frac{1}{2}} = \frac{16.6}{0.2712} \sqrt{h} = 61.2 \sqrt{h}$$

h in	\sqrt{h} (in) ^{1/2}	\bar{U} ft/sec	$\sqrt{\bar{U}}$ (ft/sec) ^{1/2}	I (ma)		I^2 (ma) ²	
				$\gamma=1.2$	$\gamma=1.4$	$\gamma=1.2$	$\gamma=1.4$
0.195	0.442	27.1	5.20	35.06	45.35	1229	2057
0.153	0.391	23.9	4.89	34.43	44.68	1189	1996
0.139	0.373	22.8	4.78	33.90	44.28	1149	1961
0.117	0.342	20.9	4.57	33.80	43.75	1142	1914
0.082	0.286	17.5	4.18	33.18	42.88	1101	1839
0.040	0.200	12.2	3.49	31.35	40.75	983	1661
0.000	0.000	0.0	0.00	23.25	30.32	541	919

This data is plotted in figure 8. The curves follow the general shape predicted by the King equation for hot wires:

$$I^2 = C_1 + C_2 \sqrt{U}$$

2. Determination of $I' \equiv \partial I / \partial U$

The slope of the curve of wire current versus incident velocity (I') is one of the quantities required for the solution of equation 72. Differentiating the King equation, we obtain

$$2 I \cdot \partial I = 0 + C_2 (U)^{\frac{1}{2}} \partial U$$

$$I' \equiv \left(\frac{\partial I}{\partial U} \right)_t = \frac{C_2}{2 I \sqrt{U}}$$

Typical calculations for the following cases are shown on

5/3/58 follows:

$$L = 100.12 \text{ m} \quad H = 1.2 \text{ m} \quad \theta = 27.2^\circ$$

$$P = \frac{(1.000)(1.000)(1.000)}{(1.000 + 1.000)} = \frac{1.000}{2.000} = 0.5000 \text{ (approx)}$$

$$\bar{H} = 1.0 \text{ m} \quad \left[\frac{1.0}{0.5000} \right] = \frac{2.0}{0.5000} \quad \sqrt{H} = 0.7071$$

\bar{H}	\sqrt{H}	$\frac{1}{\sqrt{H}}$	$\frac{1}{H}$	$\frac{1}{H^2}$	$\frac{1}{H^3}$	$\frac{1}{H^4}$
0.100	0.316	3.162	10.000	100.000	1000.000	10000.000
0.125	0.354	2.828	8.000	64.000	512.000	4096.000
0.150	0.387	2.582	6.667	44.444	296.300	1975.300
0.175	0.418	2.381	5.714	32.653	183.829	1023.619
0.200	0.447	2.236	5.000	25.000	125.000	62500.000
0.225	0.474	2.110	4.444	19.753	87.799	39062.500
0.250	0.500	2.000	4.000	16.000	64.000	25600.000
0.275	0.524	1.909	3.636	13.274	49.431	18015.873
0.300	0.548	1.818	3.333	11.111	37.037	12345.679
0.325	0.570	1.754	3.077	9.477	29.629	9479.012
0.350	0.592	1.693	2.857	8.163	23.438	7962.000
0.375	0.613	1.633	2.667	7.143	19.753	6790.123
0.400	0.632	1.585	2.500	6.349	15.625	5781.250
0.425	0.651	1.537	2.353	5.623	13.274	4943.100
0.450	0.669	1.490	2.222	5.000	11.111	4259.259
0.475	0.687	1.455	2.105	4.444	9.477	3675.300
0.500	0.707	1.414	2.000	4.000	8.000	3200.000

This data is plotted in figure 1. The curves define the general shape

predicted by the first equation for the second

$$\bar{H} = \frac{1}{2} + \frac{1}{2} \sqrt{1 + 4P}$$

3. Determination of \bar{H} and \bar{H}^2

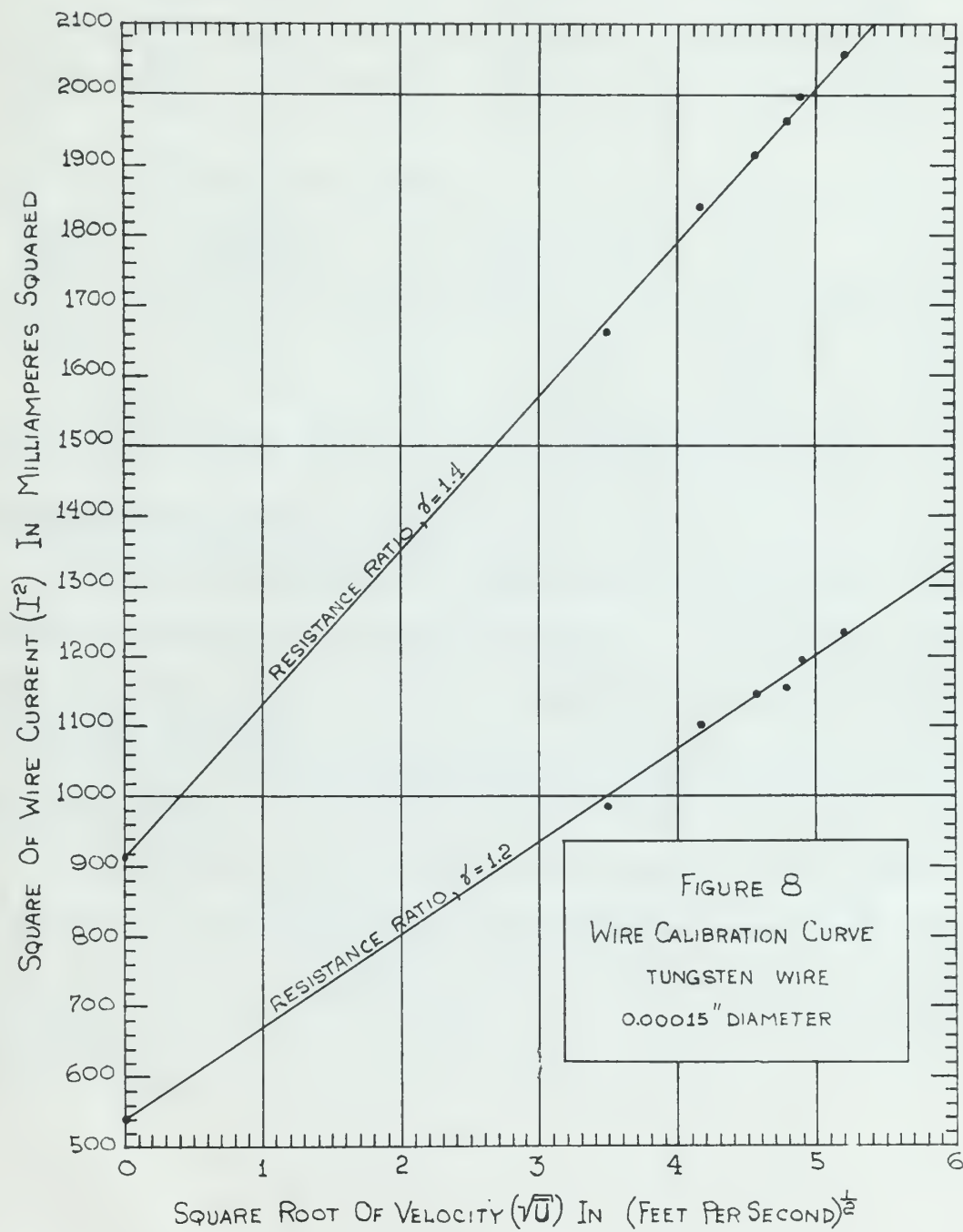
The slope of the curve of \bar{H} versus \bar{H}^2 is constant and is equal to $\frac{1}{2}$.

is not of the parabolic nature for the solution of equation 1. It is

noting the fact equation, we obtain

$$1.191 + 0 + 0.5 \bar{H}^2 = 0$$

$$\bar{H}^2 = \frac{1.191}{0.5} = 2.382$$



where C_2 is the slope of the calibration curves obtained in part 1. above. For the free stream velocity chosen for a particular experimental run, the value of (\sqrt{U}) is computed and the corresponding current (I) is read from the calibration curve. With these values, the required slope (I') is calculated. Results are shown for a tunnel velocity of twenty-six feet/second.

U ft/sec	\sqrt{U} (ft/sec) $^{\frac{1}{2}}$	γ	C_3 (ma) 2 /ft/sec) $^{\frac{1}{2}}$	I ma	I' ma/ft/sec
26.0	5.10	1.2	132.6	34.86	0.1865
		1.4	217.4	43.03	0.2366

3. Amplifier Compensation

As discussed in appendix C, a square wave is employed as a reference voltage to determine the anemometer's response to a fluctuating voltage across the wire. The HEB amplifier compensation is set by observing on an oscilloscope the waveform which results when a square wave is impressed on the wire circuit. An undistorted waveform indicates the proper calibration.

To determine the effect of compensation setting on the voltmeter output, measurements were made in the free stream with the hot wire oriented vertically. The values of M_{90} and M_{90c} were recorded as the compensation was varied.

where C_2 is the slope of the calibration curves obtained in part I, above. For the time across relay, chosen for a particular experimental run, the value of (\sqrt{V}) is computed and the corresponding current (I) is read from the calibration curve. With these values, the required slope (I') is computed. Results are shown for a number of values of frequency in kilocycles.

I' ma/sec	I ma	C_2 (ma) ² /10/sec ²	γ	\sqrt{V} (sec) ^{1/2}	V sec
0.1000	34.40	124.8	1.1	2.10	20.0
0.2500	42.63	217.6	1.4		

3. Miller Compensation

As discussed in Appendix C, a square wave is employed as a reference voltage to determine the instrument's response to a fluctuating voltage across the wire. The only required compensation is set by observing on an oscilloscope the waveforms which result when a square wave is impressed on the wire circuit. An undistorted waveform indicates the proper calibration. To determine the effect of compensation setting on the voltmeter output, measurements were made for the four settings with the test circuit as previously varied. The values of R_{eq} and R_{int} were recorded as the compensation was varied.

Dial Setting	M_{on} (mv)	M_{ecn} (mv)	M_{sn}^2	M_{ben}^2	M_n^2	M_s^2	M_c^2	$(\frac{M_s}{M_c})^2$
60	12.8	47.3	164	2230	64	100	2066	0.0484
80	13.3	55.2	176	3040	64	112	2864	0.0391
100	15.6	64.7	242	4170	64	178	3928	0.0454
125	16.2	74.0	262	5460	64	198	5198	0.0381
150	17.2	84.0	296	7050	64	232	6754	0.0344
200	22.2	126	492	15650	64	428	15358	0.0278

This data is plotted in figure 9. The response is approximately linear with no sharp null or peak; hence, proper calibration is difficult. It is estimated that the error introduced in the quantity $(M_s/M_c)^2$ when setting the compensation is of the order of 8.2% when the dial is varied by one dial unit.

4. Square Wave Current

Reference 12 gives the following formula for computing the square current of the HMB:

$$i_s = \frac{3.05}{1 + \frac{(BN)(\gamma)}{400}} = \frac{1220}{400 + (BN)(\gamma)} = \frac{1220}{D}$$

where (BN) is the bridge null setting. It is important to maintain the battery supplying this current near its rated voltage.

Weight (mg)	$\frac{1}{\text{Weight}}$ (mg ⁻¹)	$\frac{1}{\text{Weight}}$ (mg ⁻¹)	$\frac{1}{\text{Weight}}$ (mg ⁻¹)	$\frac{1}{\text{Weight}}$ (mg ⁻¹)	$\frac{1}{\text{Weight}}$ (mg ⁻¹)	$\frac{1}{\text{Weight}}$ (mg ⁻¹)	$\frac{1}{\text{Weight}}$ (mg ⁻¹)	$\frac{1}{\text{Weight}}$ (mg ⁻¹)
80	0.0125	0.0125	0.0125	0.0125	0.0125	0.0125	0.0125	0.0125
80	0.0125	0.0125	0.0125	0.0125	0.0125	0.0125	0.0125	0.0125
100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100
125	0.0080	0.0080	0.0080	0.0080	0.0080	0.0080	0.0080	0.0080
150	0.0067	0.0067	0.0067	0.0067	0.0067	0.0067	0.0067	0.0067
200	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050

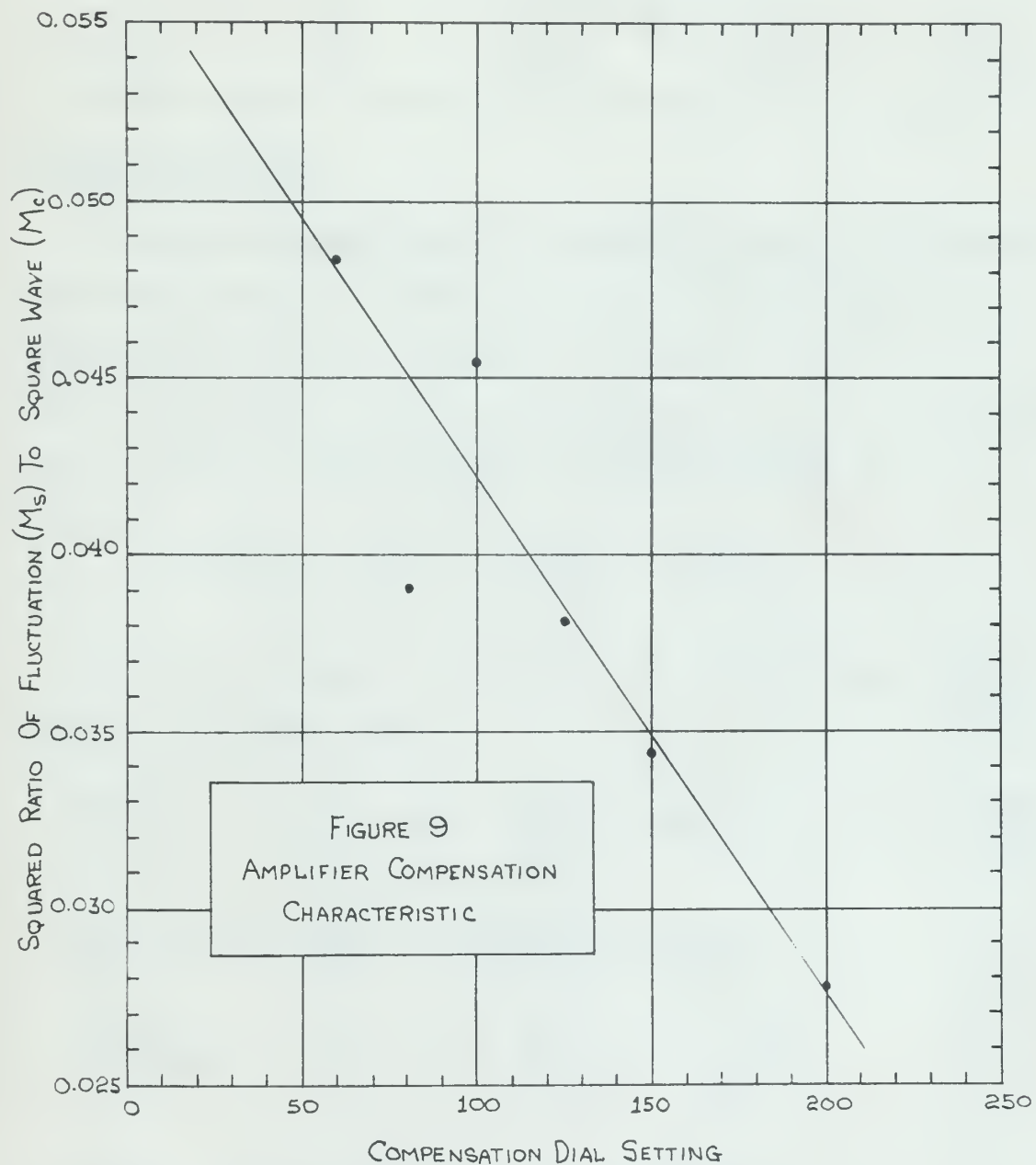
This data is plotted in Figure 6. The response is approximately linear with no sharp fall or peak. Hence, proper calibration is sufficient. It is indicated that the error introduced in the quantity $(Q/V_0)^{1/2}$ when using the composition is of the order of 0.1% when the data is varied by one data point.

4. POLYMER FILM THICKNESS

Relationship is given the following formula for computing the square content of the film:

$$S = \frac{1.00}{1 + \frac{1.00}{(Q/V_0)^{1/2}}} = \frac{1.00}{1 + \frac{1.00}{(Q/V_0)^{1/2}}}$$

where S is the polymer film thickness. It is important to indicate the polymer applying this method from the varied volume.



γ	BN	$(\gamma)(BN)$	D	i_s (ma)
1.2	477	572.4	972.4	1.255
1.4	477	667.8	1067.8	1.142

5. Temperature Profile

The pertinent formula for calculating the difference (ΔT) between the temperature (T_g) at a point in the thermal boundary layer and the stream ambient temperature is derived as follows:

$$F(pV) = \frac{I_1^2 R_o \alpha}{L} \left(\frac{R}{R-R_g} \right)^2 = \frac{I_2^2 R_o \alpha}{L} \left(\frac{R}{R-R_e} \right)^2$$

$$\text{from which, } \left(\frac{I_1}{I_2} \right)^2 = \frac{R - R_g}{R - R_e} = \frac{T - \bar{T}_g}{T - T_e}$$

where (I_1) is the wire current for a cold plate and (I_2) is that current for the heated plate.

$$\text{Taking } R = R_g \left[1 + \alpha_g (T - \bar{T}_g) \right] \text{ and rearranging, } T = \bar{T}_g + \left(\frac{\gamma-1}{\alpha_g} \right)$$

$$\text{substituting, } \left(\frac{I_1}{I_2} \right)^2 = \frac{[(\gamma-1)/\alpha_g]}{[(\gamma-1)/\alpha_g] + (\bar{T}_g - T_e)}$$

$$\text{Rearranging, } (\bar{T}_g - T_e) = \frac{\gamma-1}{\alpha_g} \left[1 - \left(\frac{I_1}{I_2} \right)^2 \right];$$

$$\text{taking, } \alpha_g = \frac{\alpha_e}{1 + \alpha_e (\bar{T}_g - T_e)} = \frac{\alpha_e}{1 + \alpha_e (\Delta T)}$$

γ	m	$(\gamma)(m)$	D	λ (cm)
1.2	477	572.4	872.4	1.222
1.4	477	667.8	1067.8	1.142

2. Temperature Profile

The pertinent formula for calculating the difference (ΔT) between

the temperature (T_s) at a point in the central boundary layer and the

stream ambient temperature is derived as follows:

$$\begin{aligned} \theta(\eta) &= \frac{I_1}{I_2} \frac{R}{L} \left(\frac{a}{R-a} \right) \left(\frac{1}{R-a} \right) = \frac{I_1}{I_2} \frac{R}{L} \left(\frac{a}{R-a} \right) \left(\frac{R}{R-a} \right) \\ \text{from which, } \left(\frac{I_1}{I_2} \right) &= \frac{R-a}{R} = \frac{T - T_s}{T - T_a} \end{aligned}$$

where (I_1) is the wire current for a cold plate and (I_2) is the

current for the heated plate.

Taking $R = R_0 [1 + \alpha (T - T_0)]$ and rearranging, $T = \bar{T}_s + \left(\frac{\gamma-1}{\alpha} \right)$

$$\text{substituting, } \left(\frac{I_1}{I_2} \right)^2 = \frac{[(\gamma-1)\alpha]}{[(\gamma-1)\alpha] + (T_s - T_0)}$$

$$\text{Rearranging, } (\bar{T}_s - T_0) = \frac{\gamma-1}{\alpha} \left[1 - \left(\frac{I_1}{I_2} \right)^2 \right]$$

$$\text{Taking, } \alpha = \frac{1}{1 + \alpha (T_s - T_0)} = \frac{1}{1 + \alpha (\Delta T)}$$

and substituting, it can be shown that

$$(T_g - T_e) = \frac{1}{\alpha_o \left[\frac{1}{(\delta - 1) \left[1 - (I_1/I_2)^2 \right]} - 1 \right]} \quad (73)$$

For an ambient temperature of 24.8°C , the coefficient (α_e) is computed as follows:

$$\alpha_e = \frac{\alpha_o}{1 + \alpha_o(T_e - T_o)} = \frac{0.00454}{1 + 0.00454(24.8^\circ - 20^\circ)} = 0.00444$$

Since the wire is more sensitive to temperature variations at low currents, the data for the temperature profile was taken at a resistance ratio of 1.1. That data and the values computed with equation 73 are shown in the table below and plotted in figure 10.

y (in)	I_1 (ma)	I_2 (ma)	$(\bar{T}_g - T_e)$ ($^\circ\text{C}$)	$\frac{(\bar{T}_g - T_e)}{T_g - T_e}$ ($^\circ\text{C}$)
0.068"	20.91	22.51	3.13°	0.391
0.118"	21.06	24.05	3.78°	0.472
0.168"	22.41	23.42	1.92°	0.240
0.218"	22.75	23.51	1.45°	0.181
0.268"	23.06	23.66	1.13°	0.141
0.368"	23.35	23.74	0.73°	0.091
0.468"	23.92	23.85	-0.14°	-0.018

The normalized temperature profile is plotted in figure 2 of Section III.

and substituted in eq. (1) to give

$$(1) \quad \frac{1}{\alpha} = \frac{1}{\alpha_0} + \frac{1}{\alpha_0} \left[\frac{(\delta - \alpha_0)}{(\delta - \alpha_0)} \right]$$

For an initial concentration of 36.5% the calculated α is

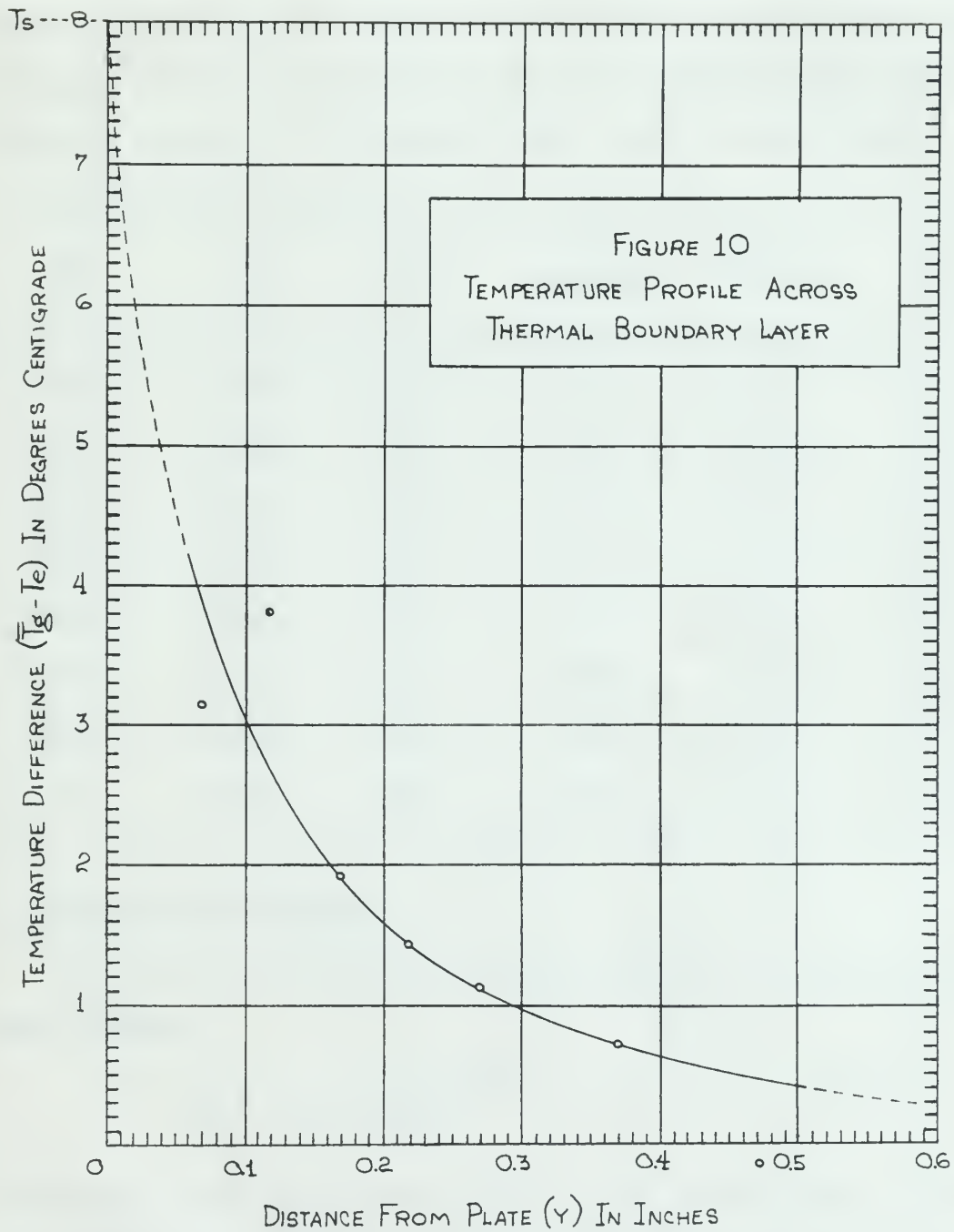
calculated as follows:

$$\alpha = \frac{1}{1 + \frac{1}{\alpha_0} \left[\frac{(\delta - \alpha_0)}{(\delta - \alpha_0)} \right]} = 0.0001$$

Since the value of α is very small, the temperature variation of α is negligible, and the data for the temperature variation of α is taken as 1. The data for the temperature variation of α is shown in Figure 1. The data for the temperature variation of α is shown in Figure 1.

$\frac{(\delta - \alpha_0)}{(\delta - \alpha_0)}$	$\frac{(\delta - \alpha_0)}{(\delta - \alpha_0)}$	$\frac{(\delta - \alpha_0)}{(\delta - \alpha_0)}$	$\frac{(\delta - \alpha_0)}{(\delta - \alpha_0)}$	$\frac{(\delta - \alpha_0)}{(\delta - \alpha_0)}$
0.000	1.000	1.000	1.000	1.000
0.001	1.000	1.000	1.000	1.000
0.002	1.000	1.000	1.000	1.000
0.003	1.000	1.000	1.000	1.000
0.004	1.000	1.000	1.000	1.000
0.005	1.000	1.000	1.000	1.000
0.006	1.000	1.000	1.000	1.000
0.007	1.000	1.000	1.000	1.000
0.008	1.000	1.000	1.000	1.000
0.009	1.000	1.000	1.000	1.000

The calculated temperature variation of α is shown in Figure 1.



6. Velocity Profile

The velocity profile across the momentum boundary layer is obtained with the plate cold and the wire oriented vertically. The values of current recorded are then used as arguments to enter figure 8 and determine the velocity. The data and results are shown below together with a plot of the profile in figure 11. The normalized plot appears in the RESULTS.

y (in)	I (ma)	I^2 (ma) ²	$\sqrt{\bar{U}}$ (ft/sec) ^{1/2}	\bar{U} (ft/sec)	$\frac{\bar{U}}{U_\infty}$
0.068"	44.00	1936	4.63	21.44	0.758
0.118"	44.35	1967	4.78	22.85	0.807
0.168"	44.48	1978	4.82	23.23	0.821
0.218"	44.68	1996	4.95	24.50	0.866
0.268"	44.78	2005	4.99	24.90	0.880
0.368"	44.98	2023	5.04	25.40	0.898
0.468"	45.12	2036	5.10	26.01	0.920
ϕ	45.70	2088	5.32	28.30	1.000

7. Boundary Layer Thickness

The following equation may be used to predict the momentum boundary layer thickness,

$$\delta_m = 0.37x \left(\frac{v_\infty x}{\nu} \right)^{-1/5}$$

Considering a point twenty-nine inches from the leading edge of the plate and a free stream velocity of twenty-six feet per second, the thickness will be,

6. Velocity Profile

The velocity profile across the channel boundary layer is obtained with the plate coils and the wire stretched vertically. The values of constant resistance are then used as arguments to enter Figure 5 and determine the velocity. The data and results are then plotted together with a plot of the profile in Figure 11. The normalizing plot appears in the results.

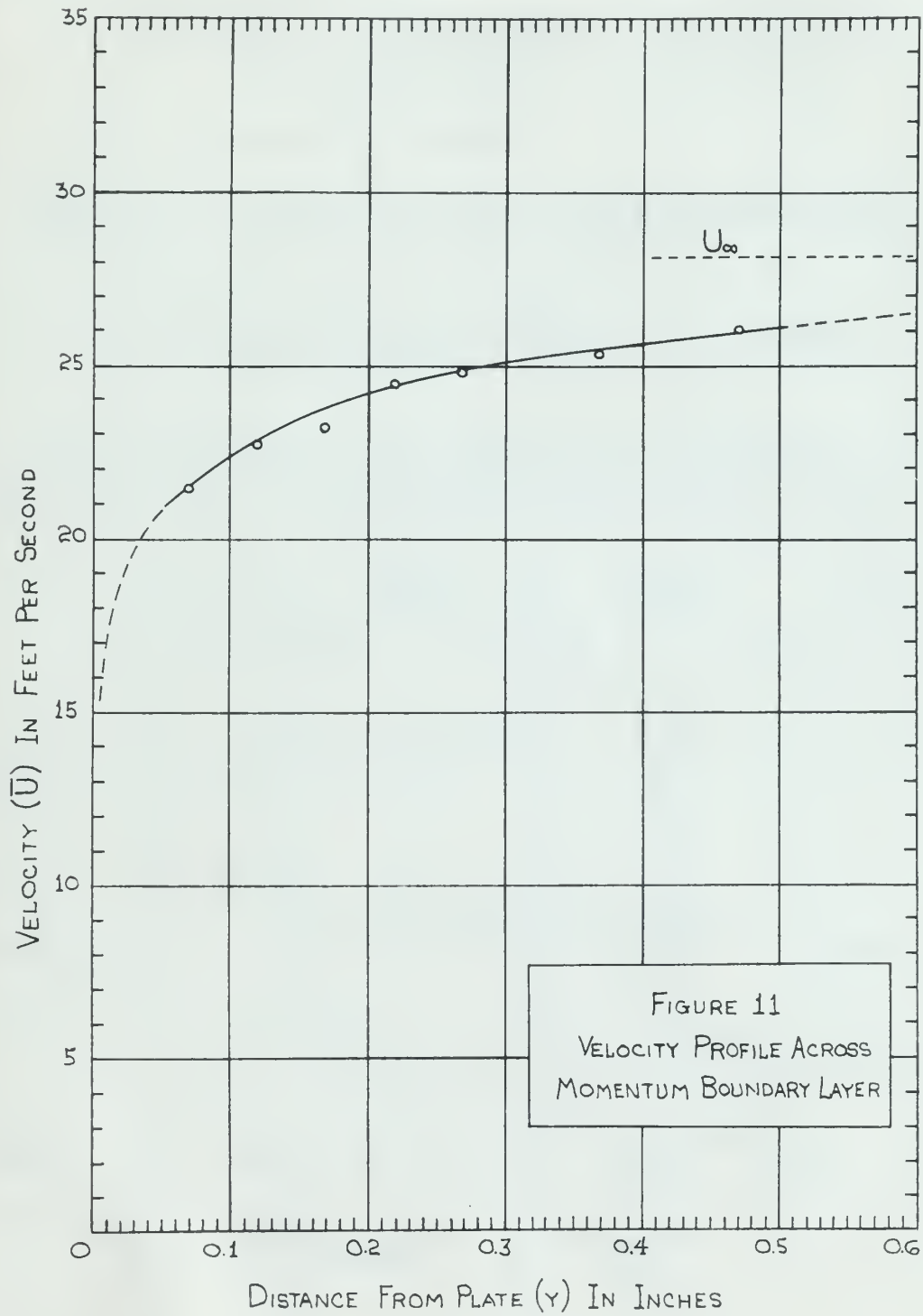
$\frac{y}{\delta}$ (in)	$\frac{U}{U_{\infty}}$ (sec)	$\sqrt{\frac{\tau}{\rho}}$ (1/2 sec) ^{1/2}	$\frac{U}{U_{\infty}}$ (sec)	$\frac{y}{\delta}$ (in)	$\frac{U}{U_{\infty}}$ (sec)
0.000	14.00	1.93	1934	17.40	0.778
0.110	14.32	1.78	1927	17.40	0.697
0.180	14.48	1.61	1979	17.40	0.621
0.210	14.58	1.52	1990	17.40	0.585
0.240	14.78	1.38	2000	17.40	0.500
0.340	14.90	1.00	1983	17.40	0.198
0.400	15.15	1.10	2026	17.40	0.320
0	15.70	1.11	2008	17.40	1.000

7. Boundary Layer Thickness

The following equation can be used to locate the boundary layer thickness,

$$\delta = 0.37x \left(\frac{\nu}{U_{\infty}} \right)^{-1/2}$$

Considering a point exactly-half inch from the leading edge of the plate and a free stream velocity of twenty-five feet per second, the thickness will be,



$$V_{\infty} = (26 \text{ ft/sec})(3600 \text{ sec/hr}) = 9.36 \times 10^4 \text{ ft/hr.}$$

$$\rho = \frac{(0.02901)(p \text{ mm Hg})}{(t^{\circ}\text{C} + 273.2^{\circ})} = \frac{(0.02901)(767.05)}{(298.0)} = 0.0747 \text{ lbs/ft}^3$$

$$\nu = \frac{\mu}{\rho} = \frac{0.460 \text{ lbs/ft-hr}}{0.0747 \text{ lbs/ft}^3} = 6.16 \frac{\text{ft}^2}{\text{hr}}$$

$$x = 29 \text{ inches} = 2.42 \text{ ft.}$$

$$\text{then } \delta_m = (0.37)(29 \text{ inches}) \left[\frac{(9.36 \times 10^4 \text{ ft/hr})(2.42 \text{ ft})}{(6.16 \text{ ft}^2/\text{hr})} \right]^{-1/5}$$

$$\delta_m = \frac{(10.73)}{(3.68 \times 10^4)^{1/5}} = \frac{(10.73)}{(8.13)} = 1.32 \text{ inches}$$

This thermal boundary layer thickness may be calculated in a similar manner employing the following equation taken from reference 14,

$$\delta_t = K_8 \left(\frac{V_{\infty} x}{\nu} \right)^{\frac{35b-36}{5(15-7b)}} \left(\frac{c_p \mu}{k} \right)^{\frac{7(c-1)}{15-7b}} (x)$$

$$\text{where, } K_8 = \left[\frac{0.37^{1/7} \times 72K}{7} \right]^{\frac{7}{15-7b}}$$

$$K = 0.0167; \quad b = 4/5; \quad c = 1/3$$

$$\text{substituting, } K_8 = \left[\frac{(0.37^{1/7})(72)(0.0167)}{7} \right]^{\frac{7}{15-5.6}} = 0.243$$

$$Pr = \frac{c_p \mu}{k} = 0.71 \quad \text{and } x = 10 \text{ inches}$$

$$\text{then, } \delta_t = 0.243 (3.68 \times 10^4)^{-0.1701} (0.71)^{-0.497} (10)$$

$$\delta_t = \frac{(2.43)}{(5.96)(0.844)} = 0.483 \text{ inches}$$

$$V_{90} = (20 \text{ ft/sec})(1000 \text{ sec/hr}) = 2.0 \times 10^4 \text{ ft/hr.}$$

$$\rho = \frac{(0.025)(0.01 \text{ ft})}{(0.01 + 0.01) \text{ ft}} = \frac{(0.00025)(0.01 \text{ ft})}{(0.02 \text{ ft})} = 0.000125 \text{ lb/ft}^3$$

$$\gamma = \frac{M}{\rho} = \frac{0.000125 \text{ lb/ft}^3}{0.01 \text{ ft}} = 0.0125 \frac{\text{lb}}{\text{ft}^2}$$

$$x = 20 \text{ inches} = 2.0 \text{ ft.}$$

$$\text{then } \delta_m = (0.37)(20 \text{ inches}) \left[\frac{(0.025 \times 10^{-4} \text{ ft})(0.01 \text{ ft})}{(1.12 \text{ ft}^2/\text{hr})} \right]^{-1/2}$$

$$\delta_m = \frac{(10.77)}{(0.01 \times 10^{-4})^{1/2}} = \frac{10.77}{(0.01)} = 1.077 \text{ inches}$$

This thermal boundary layer thickness may be calculated in a similar

manner employing the following equation taken from reference 14,

$$\delta_t = \frac{K}{V} \left(\frac{h_0 x}{k} \right)^{1/2} \quad (1)$$

$$\text{where, } K = \left[\frac{0.37}{1} \times \frac{1}{1 - 0.7} \right]^{1/2}$$

$$K = 0.607, \quad h_0 = 10, \quad c = 1$$

$$\text{substituting, } \delta_t = \left[\frac{(0.37)(10)(0.01 \text{ ft})}{(1 - 0.7)} \right]^{1/2} = 0.242$$

$$\delta_{tc} = \frac{M}{K} = 0.71 \text{ and } x = 10 \text{ inches}$$

$$\text{then, } \delta_t = 0.242 (2.0 \text{ ft}) = 0.484 \text{ ft} \quad (15)$$

$$\delta_t = \frac{(0.484)(0.025)}{(0.01)} = 0.484 \text{ inches}$$

8. uv Correlation in "Cold" Momentum Boundary Layer

In the cold plate condition, equation 7 may be used to calculate the quantity \overline{uv} .

$$\overline{uv} = - \frac{1}{2(I')^2} \left[\left(\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right)_b - \left(\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right)_a \right]$$

Data was taken at two resistance ratios and averaged. The values of (I') were taken from section (b) of this appendix, while i_s was calculated for a bridge null setting of 483 by the method of section 4. The remaining data is shown in tabular form together with the results obtained, which are plotted in the RESULTS.

$\gamma = 1.2$						averaged values	
y (in)	$(M_{sn})_a$ (mv)	$(M_{scn})_a$ (mv)	(M_n) (mv)	$(M_{sn})_b$ (mv)	$(M_{scn})_b$ (mv)	\overline{uv} (ft/sec) ²	$\frac{\overline{uv}}{U_\infty^2}$
0.068	24.2	61.2	7.50	31.0	62.8	-2.75	-0.00407
0.118	23.4	61.3	"	29.2	63.3	-2.48	-0.00367
0.168	21.8	61.2	"	27.3	62.6	-2.25	-0.00333
0.218	20.8	60.7	"	25.7	61.0	-1.82	-0.00269
0.268	19.4	60.2	"	24.8	61.5	-1.40	-0.00207
0.368	19.1	59.7	"	23.3	62.4	-1.29	-0.00191
0.468	17.0	59.0	"	19.6	61.8	-0.929	-0.00137

$\gamma = 1.4$					
0.068	60.9	128.0	7.45	79.5	137.0
0.118	59.0	128.5	"	73.4	126.5
0.168	53.5	125.6	"	72.5	131.0
0.218	50.3	124.0	"	66.3	129.5
0.268	47.8	123.0	"	60.2	130.5
0.368	45.5	122.5	"	60.3	127.0
0.468	40.2	122.3	"	54.5	124.0

3. The calculation of the average resistance factor

In the case of the calculation, equation 7 may be used to calculate the

average factor

$$\bar{R} = \frac{1}{n} \sum_{i=1}^n \left(\frac{R_i}{R_{avg}} \right)^2 = \frac{1}{n} \sum_{i=1}^n \left(\frac{R_i}{R_{avg}} \right)^2$$

Data was taken at two resistance ratios and averaged. The values of (1') were taken from section (b) of this appendix, while 1' was calculated for a bridge built using of ASD by the method of section A. The resulting data is shown in Exhibit form together with the results obtained, which are plotted in the results.

$\gamma = 1.3$							
\bar{R} (in)	\bar{R}^2 (in) ²	\bar{R} (in)	\bar{R}^2 (in) ²	\bar{R} (in)	\bar{R}^2 (in) ²	\bar{R} (in)	\bar{R}^2 (in) ²
0.002	4.0E-6	0.002	4.0E-6	0.002	4.0E-6	0.002	4.0E-6
0.110	0.0121	0.110	0.0121	0.110	0.0121	0.110	0.0121
0.160	0.0256	0.160	0.0256	0.160	0.0256	0.160	0.0256
0.210	0.0441	0.210	0.0441	0.210	0.0441	0.210	0.0441
0.260	0.0676	0.260	0.0676	0.260	0.0676	0.260	0.0676
0.310	0.0961	0.310	0.0961	0.310	0.0961	0.310	0.0961
0.360	0.1296	0.360	0.1296	0.360	0.1296	0.360	0.1296
0.410	0.1681	0.410	0.1681	0.410	0.1681	0.410	0.1681
0.460	0.2116	0.460	0.2116	0.460	0.2116	0.460	0.2116
0.510	0.2601	0.510	0.2601	0.510	0.2601	0.510	0.2601
0.560	0.3136	0.560	0.3136	0.560	0.3136	0.560	0.3136
0.610	0.3721	0.610	0.3721	0.610	0.3721	0.610	0.3721
0.660	0.4356	0.660	0.4356	0.660	0.4356	0.660	0.4356
0.710	0.5041	0.710	0.5041	0.710	0.5041	0.710	0.5041
0.760	0.5776	0.760	0.5776	0.760	0.5776	0.760	0.5776
0.810	0.6561	0.810	0.6561	0.810	0.6561	0.810	0.6561
0.860	0.7396	0.860	0.7396	0.860	0.7396	0.860	0.7396
0.910	0.8281	0.910	0.8281	0.910	0.8281	0.910	0.8281
0.960	0.9216	0.960	0.9216	0.960	0.9216	0.960	0.9216
1.010	1.0201	1.010	1.0201	1.010	1.0201	1.010	1.0201
1.060	1.1236	1.060	1.1236	1.060	1.1236	1.060	1.1236
1.110	1.2321	1.110	1.2321	1.110	1.2321	1.110	1.2321
1.160	1.3456	1.160	1.3456	1.160	1.3456	1.160	1.3456
1.210	1.4641	1.210	1.4641	1.210	1.4641	1.210	1.4641
1.260	1.5876	1.260	1.5876	1.260	1.5876	1.260	1.5876
1.310	1.7161	1.310	1.7161	1.310	1.7161	1.310	1.7161
1.360	1.8496	1.360	1.8496	1.360	1.8496	1.360	1.8496
1.410	1.9881	1.410	1.9881	1.410	1.9881	1.410	1.9881
1.460	2.1316	1.460	2.1316	1.460	2.1316	1.460	2.1316
1.510	2.2801	1.510	2.2801	1.510	2.2801	1.510	2.2801
1.560	2.4336	1.560	2.4336	1.560	2.4336	1.560	2.4336
1.610	2.5921	1.610	2.5921	1.610	2.5921	1.610	2.5921
1.660	2.7556	1.660	2.7556	1.660	2.7556	1.660	2.7556
1.710	2.9241	1.710	2.9241	1.710	2.9241	1.710	2.9241
1.760	3.0976	1.760	3.0976	1.760	3.0976	1.760	3.0976
1.810	3.2761	1.810	3.2761	1.810	3.2761	1.810	3.2761
1.860	3.4596	1.860	3.4596	1.860	3.4596	1.860	3.4596
1.910	3.6481	1.910	3.6481	1.910	3.6481	1.910	3.6481
1.960	3.8416	1.960	3.8416	1.960	3.8416	1.960	3.8416
2.010	4.0401	2.010	4.0401	2.010	4.0401	2.010	4.0401
2.060	4.2436	2.060	4.2436	2.060	4.2436	2.060	4.2436
2.110	4.4521	2.110	4.4521	2.110	4.4521	2.110	4.4521
2.160	4.6656	2.160	4.6656	2.160	4.6656	2.160	4.6656
2.210	4.8841	2.210	4.8841	2.210	4.8841	2.210	4.8841
2.260	5.1076	2.260	5.1076	2.260	5.1076	2.260	5.1076
2.310	5.3361	2.310	5.3361	2.310	5.3361	2.310	5.3361
2.360	5.5696	2.360	5.5696	2.360	5.5696	2.360	5.5696
2.410	5.8081	2.410	5.8081	2.410	5.8081	2.410	5.8081
2.460	6.0516	2.460	6.0516	2.460	6.0516	2.460	6.0516
2.510	6.3001	2.510	6.3001	2.510	6.3001	2.510	6.3001
2.560	6.5536	2.560	6.5536	2.560	6.5536	2.560	6.5536
2.610	6.8121	2.610	6.8121	2.610	6.8121	2.610	6.8121
2.660	7.0756	2.660	7.0756	2.660	7.0756	2.660	7.0756
2.710	7.3441	2.710	7.3441	2.710	7.3441	2.710	7.3441
2.760	7.6176	2.760	7.6176	2.760	7.6176	2.760	7.6176
2.810	7.8961	2.810	7.8961	2.810	7.8961	2.810	7.8961
2.860	8.1796	2.860	8.1796	2.860	8.1796	2.860	8.1796
2.910	8.4681	2.910	8.4681	2.910	8.4681	2.910	8.4681
2.960	8.7616	2.960	8.7616	2.960	8.7616	2.960	8.7616
3.010	9.0601	3.010	9.0601	3.010	9.0601	3.010	9.0601
3.060	9.3636	3.060	9.3636	3.060	9.3636	3.060	9.3636
3.110	9.6721	3.110	9.6721	3.110	9.6721	3.110	9.6721
3.160	9.9856	3.160	9.9856	3.160	9.9856	3.160	9.9856
3.210	10.3041	3.210	10.3041	3.210	10.3041	3.210	10.3041
3.260	10.6276	3.260	10.6276	3.260	10.6276	3.260	10.6276
3.310	10.9561	3.310	10.9561	3.310	10.9561	3.310	10.9561
3.360	11.2896	3.360	11.2896	3.360	11.2896	3.360	11.2896
3.410	11.6281	3.410	11.6281	3.410	11.6281	3.410	11.6281
3.460	11.9716	3.460	11.9716	3.460	11.9716	3.460	11.9716
3.510	12.3201	3.510	12.3201	3.510	12.3201	3.510	12.3201
3.560	12.6736	3.560	12.6736	3.560	12.6736	3.560	12.6736
3.610	13.0321	3.610	13.0321	3.610	13.0321	3.610	13.0321
3.660	13.3956	3.660	13.3956	3.660	13.3956	3.660	13.3956
3.710	13.7641	3.710	13.7641	3.710	13.7641	3.710	13.7641
3.760	14.1376	3.760	14.1376	3.760	14.1376	3.760	14.1376
3.810	14.5161	3.810	14.5161	3.810	14.5161	3.810	14.5161
3.860	14.8996	3.860	14.8996	3.860	14.8996	3.860	14.8996
3.910	15.2881	3.910	15.2881	3.910	15.2881	3.910	15.2881
3.960	15.6816	3.960	15.6816	3.960	15.6816	3.960	15.6816
4.010	16.0801	4.010	16.0801	4.010	16.0801	4.010	16.0801
4.060	16.4836	4.060	16.4836	4.060	16.4836	4.060	16.4836
4.110	16.8921	4.110	16.8921	4.110	16.8921	4.110	16.8921
4.160	17.3056	4.160	17.3056	4.160	17.3056	4.160	17.3056
4.210	17.7241	4.210	17.7241	4.210	17.7241	4.210	17.7241
4.260	18.1476	4.260	18.1476	4.260	18.1476	4.260	18.1476
4.310	18.5761	4.310	18.5761	4.310	18.5761	4.310	18.5761
4.360	19.0096	4.360	19.0096	4.360	19.0096	4.360	19.0096
4.410	19.4481	4.410	19.4481	4.410	19.4481	4.410	19.4481
4.460	19.8916	4.460	19.8916	4.460	19.8916	4.460	19.8916
4.510	20.3401	4.510	20.3401	4.510	20.3401	4.510	20.3401
4.560	20.7936	4.560	20.7936	4.560	20.7936	4.560	20.7936
4.610	21.2521	4.610	21.2521	4.610	21.2521	4.610	21.2521
4.660	21.7156	4.660	21.7156	4.660	21.7156	4.660	21.7156
4.710	22.1841	4.710	22.1841	4.710	22.1841	4.710	22.1841
4.760	22.6576	4.760	22.6576	4.760	22.6576	4.760	22.6576
4.810	23.1361	4.810	23.1361	4.810	23.1361	4.810	23.1361
4.860	23.6196	4.860	23.6196	4.860	23.6196	4.860	23.6196
4.910	24.1081	4.910	24.1081	4.910	24.1081	4.910	24.1081
4.960	24.5966	4.960	24.5966	4.960	24.5966	4.960	24.5966
5.010	25.0901	5.010	25.0901	5.010	25.0901	5.010	25.0901
5.060	25.5886	5.060	25.5886	5.060	25.5886	5.060	25.5886
5.110	26.0921	5.110	26.0921	5.110	26.0921	5.110	26.0921
5.160	26.6006	5.160	26.6006	5.160	26.6006	5.160	26.6006
5.210	27.1141	5.210	27.1141	5.210	27.1141	5.210	27.1141
5.260	27.6326	5.260	27.6326	5.260	27.6326	5.260	27.6326
5.310	28.1561	5.310	28.1561	5.310	28.1561	5.310	28.1561
5.360	28.6846	5.360	28.6846	5.360	28.6846	5.360	28.6846
5.410	29.2181	5.410	29.2181	5.410	29.2181	5.410	29.2181
5.460	29.7566	5.460	29.7566	5.460	29.7566	5.460	29.7566
5.510	30.2961	5.510	30.2961	5.510	30.2961	5.510	30.2961
5.560	30.8406	5.560	30.8406	5.560	30.8406	5.560	30.8406
5.610	31.3861	5.610	31.3861	5.610	31.3861	5.610	31.3861
5.660	31.9366	5.660	31.9366	5.660	31.9366	5.660	31.9366
5.710	32.4901	5.710	32.4901	5.710	32.4901	5.710	32.4901
5.760	33.0466	5.760	33.0466	5.760	33.0466	5.760	33.0466
5.810	33.6061	5.810	33.6061	5.810	33.6061	5.810	33.6061
5.860	34.1686	5.860	34.1686	5.860	34.1686	5.860	34.1686
5.910	34.7341	5.910	34.7341	5.910	34.7341	5.910	34.7341
5.960	35.3026	5.960	35.3026	5.960	35.3026	5.960	35.3026
6.010	35.8741	6.010	35.8741	6.010	35.8741	6.010	35.8741
6.060	36.4486	6.060	36.4486	6.060	36.4486	6.060	36.4486
6.110	37.0261	6.110	37.0261	6.110	37.0261	6.110	37.0261
6.160	37.6066	6.160	37.6066	6.160	37.6066	6.160	37.6066
6.210	38.1891	6.210	38.1891	6.210	38.1891	6.210	38.1891
6.260	38.7736	6.260	38.7736	6.260	38.7736	6.260	38.7736
6.310	39.3601	6.310	39.3601	6.310	39.3601	6.310	39.3601
6.360	39.9486	6.360	39.9486	6.360	39.9486	6.360	39.9486
6.410	40.5391	6.410	40.5391	6.410	40.5391	6.410</	

9. \overline{uv} and \overline{vt} Correlations in Thermal Boundary Layer

On the hot plate condition, equation 72 may be used to calculate the quantities \overline{uv} and \overline{vt} . Rearranged,

$$\overline{uv} = \frac{\sqrt{2} (I_a + I_b)}{4 I'} \left[\frac{\alpha_e}{\gamma - [1 + \alpha_e (T_g - T_e)]} \right] = \frac{1}{2(I')^2} \left[\left(\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right)_b - \left(\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right)_a \right]$$

Data was taken at two resistance ratios as for the previous calculations. Great difficulty was experienced in obtaining reasonable results because of the scattering of the data. To reduce the random errors present the data was plotted in figure 12 through 14, and faired into smooth curves. The remainder of the calculations were carried out with the smoothed data. The raw data, the faired data and the final results are tabulated below. Normalized curves of the results appear in figures 1 and 3.

Raw Data							
$\gamma = 1.2$							
y (in.)	$4I_a$ (ma)	$4I_b$ (ma)	M_n (mv)	$(M_{sn})_a$ (mv)	$(M_{sn})_b$ (mv)	$(M_{scn})_a$ (mv)	$(M_{scn})_b$ (mv)
0.068	121.0	127.8	7.5	41.7	60.1	61.7	76.5
0.118	121.1	129.9	"	41.2	50.2	61.0	70.2
0.168	125.0	132.2	"	34.3	39.9	58.7	62.6
0.218	126.1	132.3	"	28.1	33.5	56.5	53.7
0.268	128.0	131.8	"	25.1	29.5	53.7	57.5
0.368	128.8	132.9	"	19.7	24.4	52.2	53.5
0.468	130.1	132.8	"	17.7	21.9	52.2	53.0

9. uv and its Correlation to Thermal Boundary Layer

On the hot plate condition, equation 2 may be used to calculate

the quantities uv and w . Rearranged,

$$uv = \frac{\sqrt{2} (I_u + I_p)}{2 I_p} \left[\frac{1}{1 + \frac{I_u}{I_p}} \right] = \frac{1}{2} \left[\frac{I_u}{I_p} \right] \left[\frac{1}{1 + \frac{I_u}{I_p}} \right] = \frac{1}{2} \left[\frac{I_u}{I_p} \right] \left[\frac{1}{1 + \frac{I_u}{I_p}} \right]$$

Data was taken at two resistance ratios as for the previous calculations.

Great difficulty was experienced in obtaining reproducible results because of

the wandering of the data. To reduce the random errors present the data

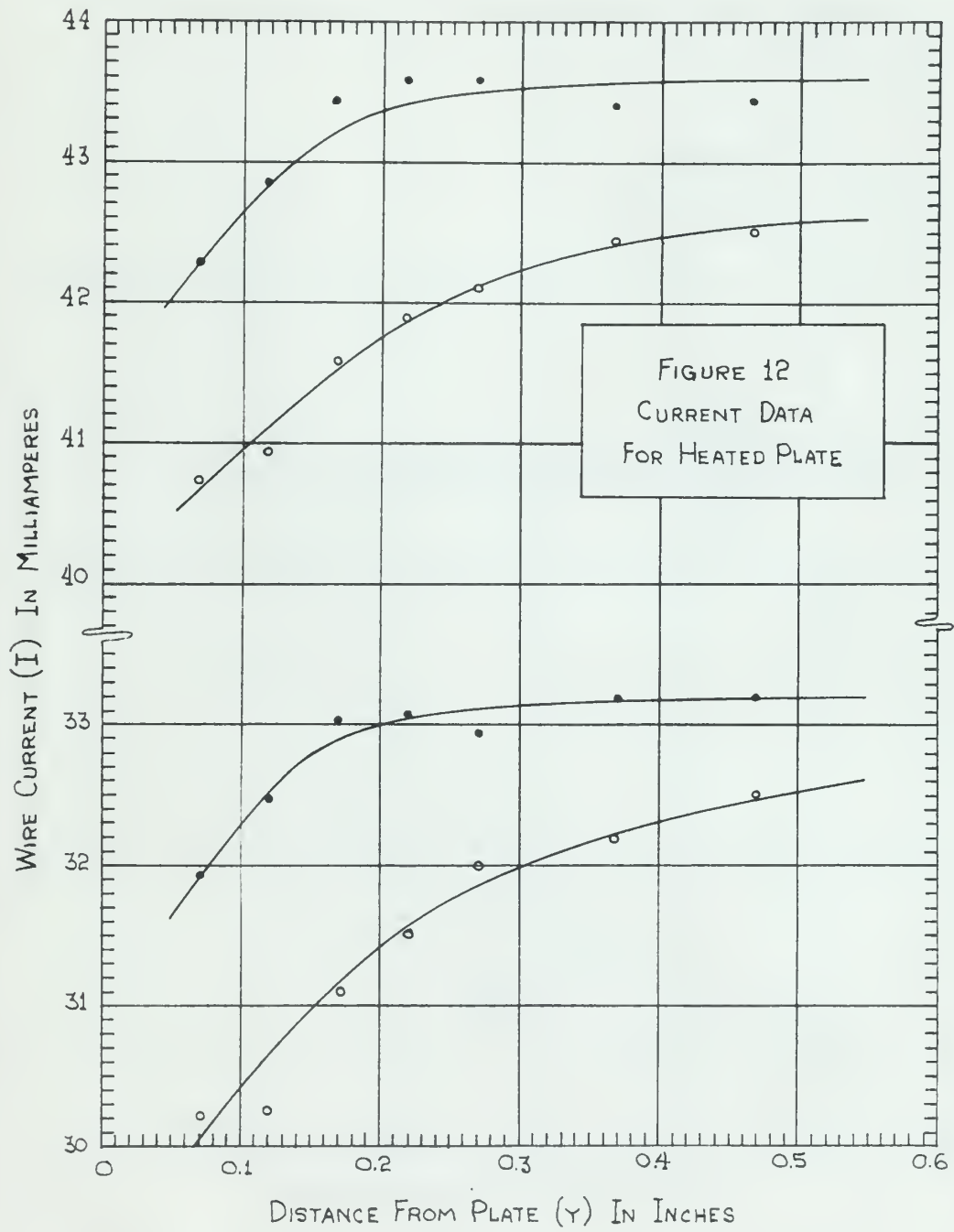
was plotted in Figure 12 through 14, and fitted into smooth curves. The

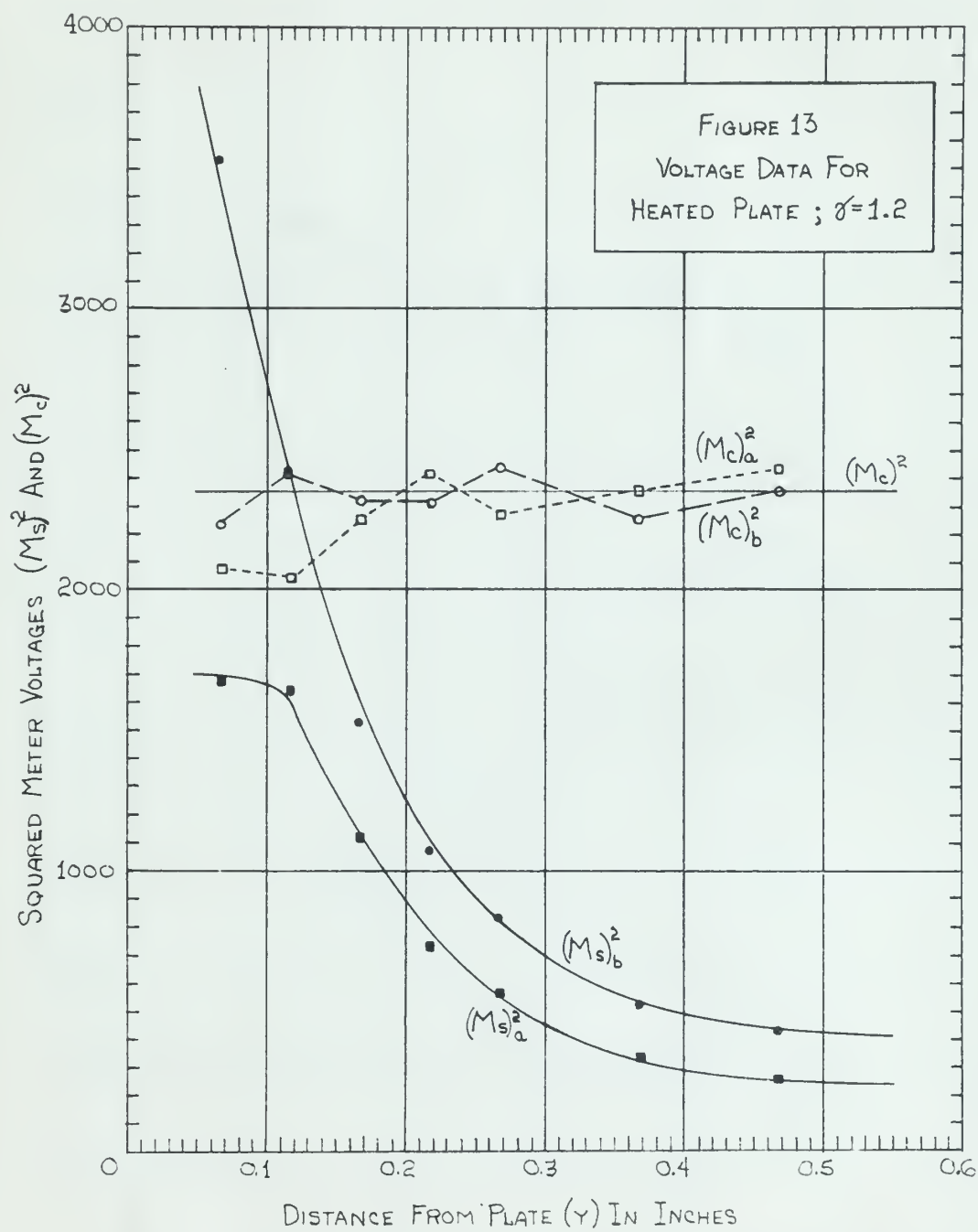
remainders of the calculations were plotted and with the smoothed data. The

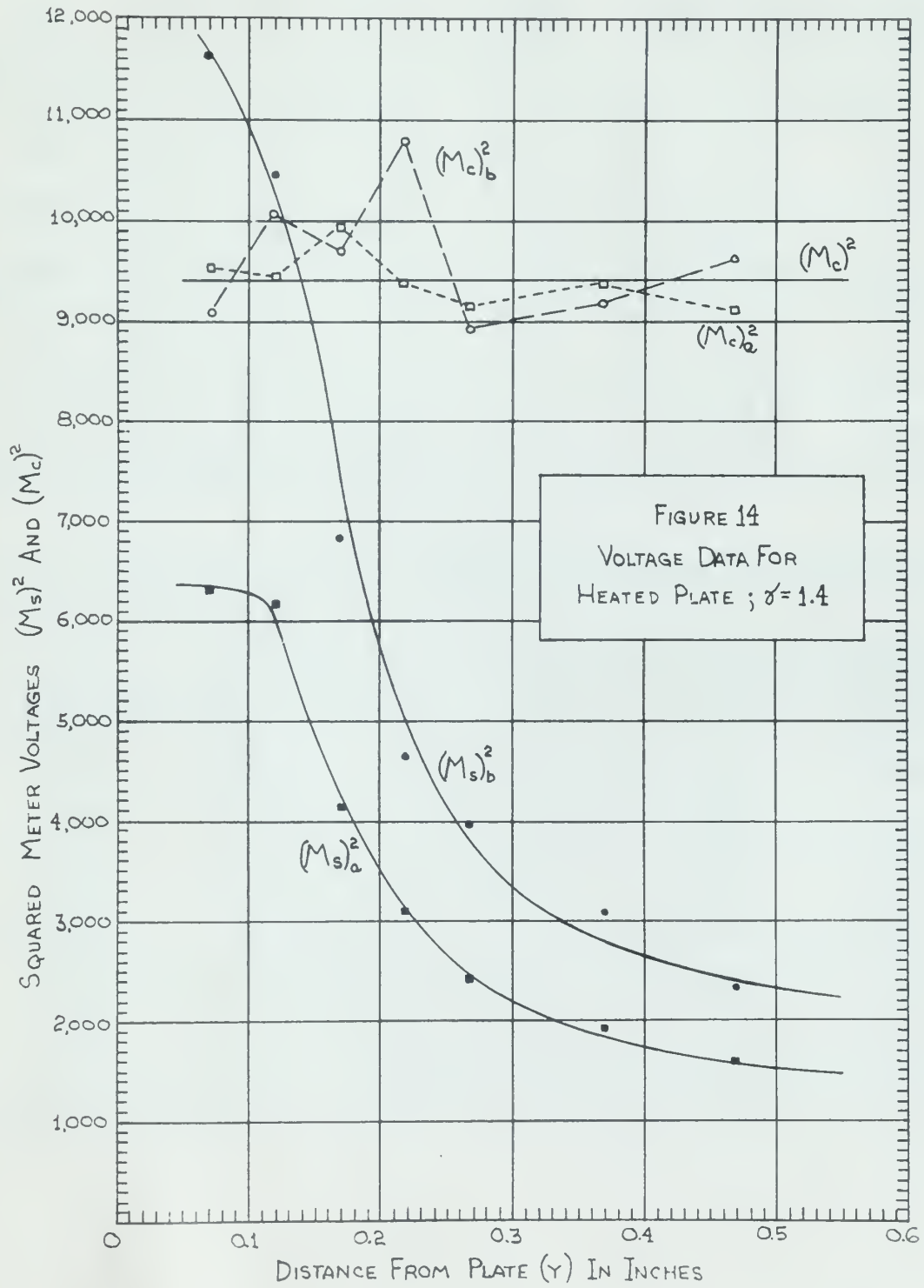
raw data, the fitted data and the final results are tabulated below. Normal-

ized curves of the results appear in Figures 1 and 3.

Raw Data							
$\gamma = 1.2$							
γ (%)	I_p (w)	I_u (w)	I_{up} (w)	I_{up} (w)	I_{up} (w)	I_{up} (w)	I_{up} (w)
0.062	121.0	127.8	7.2	41.7	69.1	67.7	70.2
0.110	121.1	127.9	"	41.2	70.2	67.0	70.2
0.168	122.0	128.2	"	36.3	70.9	70.7	63.6
0.218	126.1	128.3	"	28.1	72.2	72.2	53.7
0.266	126.0	131.8	"	22.1	74.2	72.7	57.2
0.328	128.2	132.3	"	19.7	74.2	71.2	53.2
0.461	130.1	132.8	"	17.7	71.8	72.2	53.0







Raw Data, con.

$$\gamma = 1.4$$

y (in)	$4I_a$ (ma)	$4I_b$ (ma)	M_D (mv)	$(M_{sn})_a$ (mv)	$(M_{sn})_b$ (mv)	$(M_{scn})_a$ (mv)	$(M_{scn})_b$ (mv)
0.068	163.0	169.2	7.5	79.7	108.0	126.0	144.0
0.118	163.8	171.4	"	78.5	102.5	125.0	143.5
0.168	166.3	173.3	"	64.7	83.1	119.0	129.0
0.218	167.7	174.3	"	56.0	68.6	112.0	124.5
0.268	168.5	174.3	"	49.8	63.6	108.0	114.0
0.368	169.8	173.6	"	44.5	56.0	106.5	111.0
0.468	170.0	173.7	"	41.0	49.3	104.0	110.0

Fairred Data

$$\gamma = 1.2$$

y (in)	I_a (ma)	I_b (ma)	$(M_s)_a^2$ (mv) ²	$(M_s)_b^2$ (mv) ²	$(M_c)^2$ (mv) ²
0.068	30.01	31.93	1690	3520	2350
0.118	30.65	32.51	1620	2410	"
0.168	31.18	32.89	1130	1600	"
0.218	31.60	33.06	770	1100	"
0.268	31.88	33.13	530	810	"
0.368	32.23	33.19	330	550	"
0.468	32.47	33.20	240	440	"

Raw Data, cont.

 $\gamma = 1.4$

V (mV)	I_B (mA)	I_D (mA)	M_{D1} (mV)	M_{D2} (mV)	M_{D3} (mV)	M_{D4} (mV)	M_{D5} (mV)
0.068	100.0	100.0	100.0	100.0	100.0	100.0	100.0
0.118	100.0	100.0	100.0	100.0	100.0	100.0	100.0
0.168	100.0	100.0	100.0	100.0	100.0	100.0	100.0
0.218	100.0	100.0	100.0	100.0	100.0	100.0	100.0
0.268	100.0	100.0	100.0	100.0	100.0	100.0	100.0
0.368	100.0	100.0	100.0	100.0	100.0	100.0	100.0
0.468	100.0	100.0	100.0	100.0	100.0	100.0	100.0

Raw Data

 $\gamma = 1.5$

V (mV)	I_B (mA)	I_D (mA)	M_{D1} (mV)	M_{D2} (mV)	M_{D3} (mV)	M_{D4} (mV)	M_{D5} (mV)
0.068	30.01	31.93	1000	3250	3250	3250	3250
0.118	30.02	32.71	1000	3410	3410	3410	3410
0.168	31.10	32.89	1130	1900	1900	1900	1900
0.218	31.00	33.00	1100	1100	1100	1100	1100
0.268	31.88	33.13	1230	810	810	810	810
0.368	32.23	33.19	1330	220	220	220	220
0.468	32.41	33.20	1340	440	440	440	440

Paired Data, con.

$$\gamma = 1.4$$

y (in)	I_a (ma)	I_b (ma)	$(M_s)_a^2$ (mv) ²	$(M_s)_b^2$ (mv) ²	$(M_c)^2$ (mv) ²
0.068	40.87	42.30	6320	11650	9400
0.118	41.13	42.81	6040	10250	"
0.168	41.55	43.22	4190	7380	"
0.218	41.89	43.42	3130	5000	"
0.268	42.10	43.50	2470	3840	"
0.368	42.41	43.56	1830	2790	"
0.468	42.56	43.58	1600	2400	"

Results

y (in)	\overline{uv} (ft/sec) ²	$\overline{\frac{uv}{U_\infty^2}}$	\overline{vt} (ft °C/sec)	$\overline{v_\infty(T_s - T_e)}$
0.068	+4.69	+0.00694	+7.96	+0.0383
0.118	-2.74	- 0.00405	+1.74	+0.00837
0.168	-3.35	-0.00496	+0.421	+0.00202
0.218	-1.41	-0.00209	+0.633	+0.00304
0.268	-0.641	-0.00095	+0.737	+0.00354
0.368	-0.205	-0.00030	+0.686	+0.00330
0.468	+0.002	+0.00000	+0.693	+0.00333

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λ (m)	I_a (mV)	I_p (mV)	(M_s) (mV)	(M_d) (mV)	(M_c) (mV)
0.008	40.87	43.30	0.030	11.020	0.000
0.118	41.13	43.81	0.040	10.320	0.000
0.168	41.22	43.33	0.030	9.320	0.000
0.218	41.22	43.43	0.030	8.000	0.000
0.268	43.10	43.20	0.040	3.400	0.000
0.368	43.41	43.26	0.030	3.200	0.000
0.468	43.26	43.26	0.000	3.200	0.000

Results

λ (m)	$(\frac{I_a}{I_p})$ (sec/sec)	$(\frac{M_s}{M_d})$ (sec/sec)	$(\frac{M_c}{M_d})$ (sec/sec)
0.008	+4.00	+0.0000	+0.0000
0.118	-5.24	-0.0040	+1.24
0.168	-3.32	-0.0000	+0.431
0.218	-1.41	-0.0030	+0.033
0.268	-0.941	-0.0002	+0.337
0.368	-0.302	-0.0000	+0.000
0.468	+0.000	+0.0000	+0.000

APPENDIX F

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